NELSON QMATHS 11 SPECIALIST MATHEMATICS

FULLY WORKED SOLUTIONS

Chapter 1 Vectors

Exercise 1.01 Vectors

Question 1

b, d, f, g (Each has both magnitude and direction)

Question 2

a 18 km east (magnitude $4.5 \times 4 \text{ km} = 18 \text{ km}$ direction east)

b 24 km north (magnitude $6 \times 4 \text{ km} = 24 \text{ km}$ direction north)

c 16 km south-west (magnitude $4 \times 4 \text{ km} = 16 \text{ km}$ direction south-west)

d $28 \text{ km N}30^\circ \text{ W}$ or $28 \text{ km } 330^\circ\text{T}$ (magnitude $7 \times 4 \text{ km} = 28 \text{ km}$ direction $\text{N}30^\circ \text{W}$ or 330°T)

e $20 \text{ km N}75^\circ \text{E}$ or $20 \text{ km } 075^\circ\text{T}$ (magnitude $5 \times 4 \text{ km} = 20 \text{ km}$ direction $\text{N}75^\circ \text{E}$ or 075°T)

f $14 \text{ km S}25^\circ \text{E}$ or $14 \text{ km } 155^\circ\text{T}$ (magnitude $3.5 \times 4 \text{ km} = 14 \text{ km}$ direction $\text{S}25^\circ \text{E}$ or 155°T)

Question 3

Choose an appropriate scale, such as 1 cm = 2 m.

a $\frac{5}{2}$ = 2.5 cm west

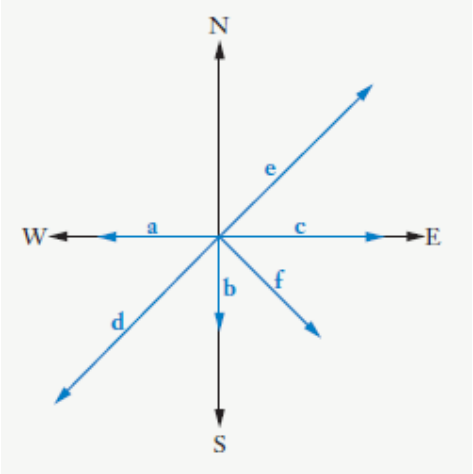
b $\frac{4}{2}$ = 2 cm south

c $\frac{7}{2}$ = 3.5 cm east

d $\frac{10}{2}$ = 5 cm south-west

e $\frac{9}{2}$ = 4.5 cm north-east

f $\frac{6}{2}$ = 3 cm south-east



Question 4

Choose an appropriate scale, such as 1 cm = 50 N.

a $\frac{200}{50}$ = 4 cm S40°W

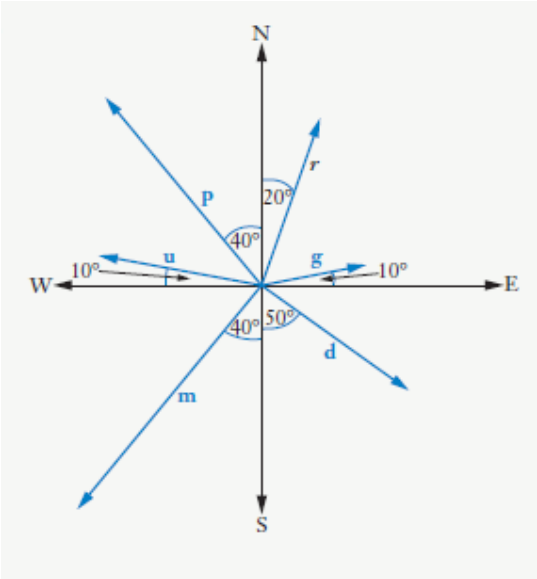
b $\frac{160}{50}$ = 3.2 cm N20°E

c $\frac{120}{50}$ = 2.4 cm 280°T

d $\frac{210}{50}$ = 4.2 cm N40°W

e $\frac{90}{50}$ = 1.8 cm 080°T

f $\frac{170}{50}$ = 3.4 cm S50°E



Question 5

Choose an appropriate scale, such as 1 cm = 2 ms^{-2}.

a $\frac{10}{2} =$ 5 cm east

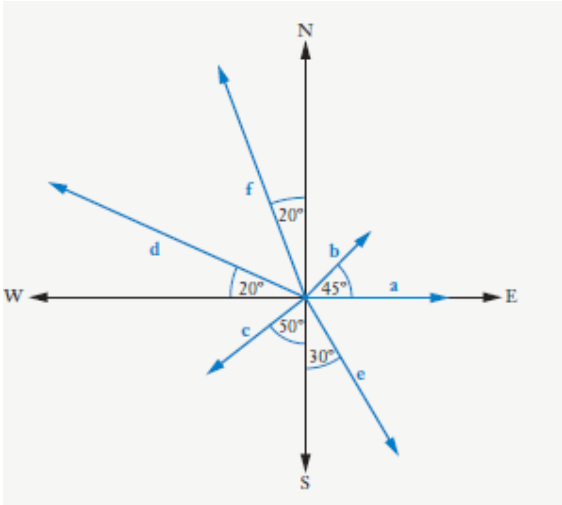
b $\frac{6}{2} =$ 3 cm north-east

c $\frac{8}{2} =$ 4 cm 230°T

d $\frac{18}{2} =$ 9 cm N70°W

e $\frac{12}{2} =$ 6 cm S30°E

f $\frac{16}{2} =$ 8 cm 340°T



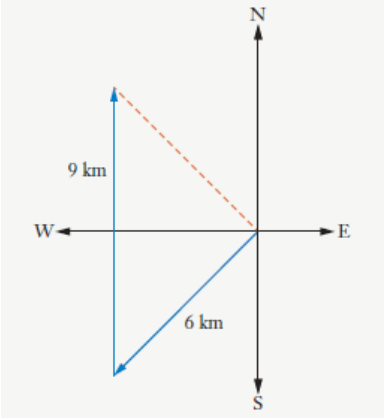
Question 6

a Choose an appropriate scale, such as 1 cm = 3 km.

The first vector is a line of length $\frac{6}{2} = 3$ cm with direction south-west.

The second vector is a line of length $\frac{9}{2} = 4.5$ cm with direction north.

Draw north–south and east–west lines and show the two vectors.



b The bearing is angle $\theta°$.

Find lengths x and y.

$x = 6\cos(45°) = 4.243$ km

$y = 6\sin(45°) = 4.243$ km

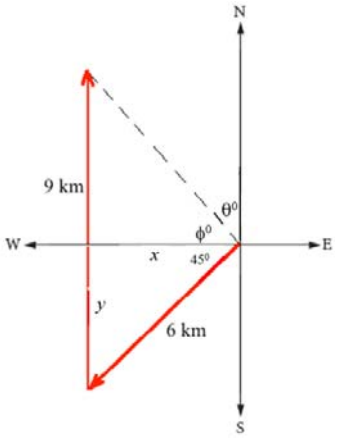
Find angle ϕ using the tan function.

$$\tan(\phi°) = \frac{9-y}{x} = 1.121$$

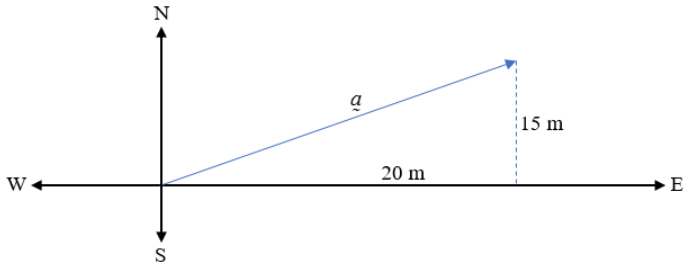
$$\phi° = \tan^{-1}(1.121) = 48.27°$$

Then $\theta = 90° - 48.27° = 41.73°$

Bearing is $= 360° - \theta = 318.27° \simeq 318°$ T



Question 7



The norm of vector $\underset{\sim}{a}$ is its magnitude.

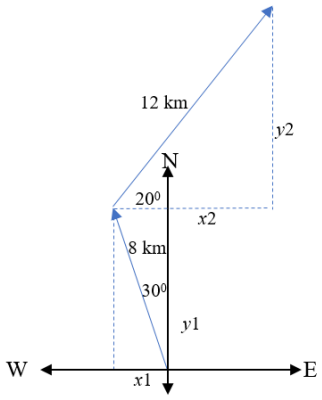
Using Pythagoras' theorem,

$$|\underset{\sim}{a}| = \sqrt{20^2 + 15^2}$$
$$= 25$$

Question 8

a

$$x_1 = 8\sin(30°) = 4.000$$
$$x_2 = 12\cos(20°) = 11.276$$
$$y_1 = 8\cos(30°) = 6.928$$
$$y_2 = 12\sin(20°) = 4.104$$

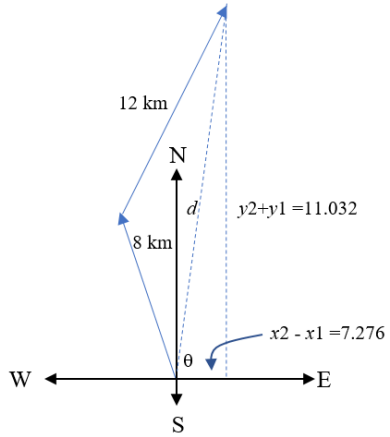


$$\tan(\theta) = \frac{11.032}{7.276} = 1.516$$

$$\theta = \tan^{-1}(1.516) = 56.6°$$

$$90 - \theta = 33.4°$$

Bearing is $033°\text{T}$



b $d = \sqrt{11.032^2 + 7.276^2} = 13.215 \text{ km}$

Exercise 1.02 Polar and Cartesian forms of vectors

Question 1

An ordered pair is written as (x, y), where x is the x-coordinate and y is the y-coordinate.

a (5, 0)

b (2, 4)

c (–4, −5)

d (0, –3)

e (–5, 4)

f (3, –5)

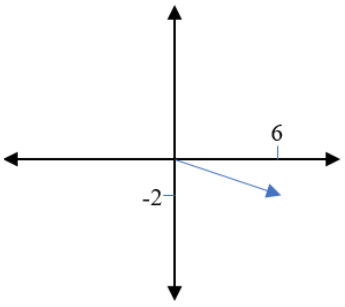
g (6, −2)

Question 2

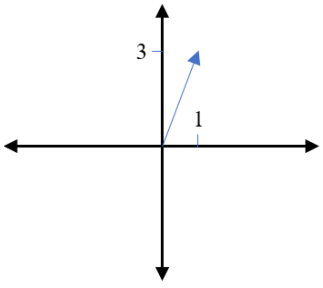
Each vector can start at any point, but it must have the direction and magnitude given.

Each solution below has the origin as the start point.

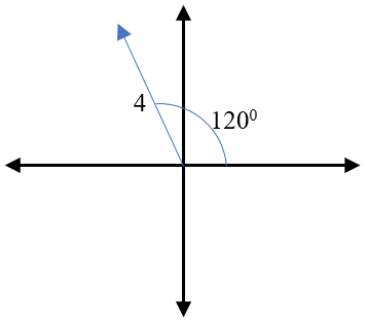
a x-coordinate is 6, y-coordinate is –2



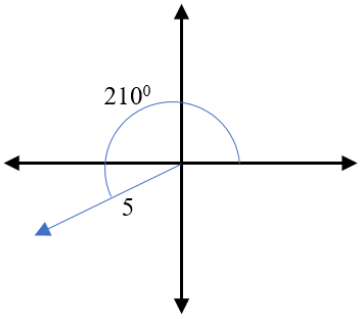
b x-coordinate is 1, y-coordinate is 3



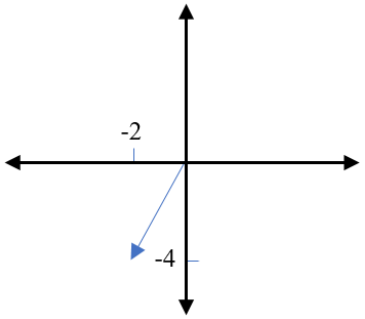
c Magnitude is 4, angle is 120° counterclockwise from the *x*-axis



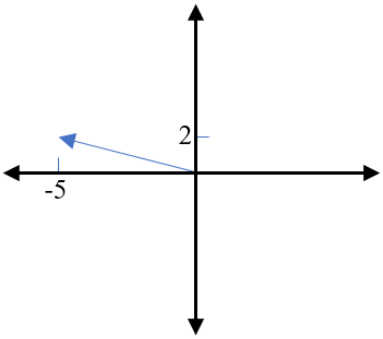
d Magnitude is 5, angle is 210° counterclockwise from the *x*-axis



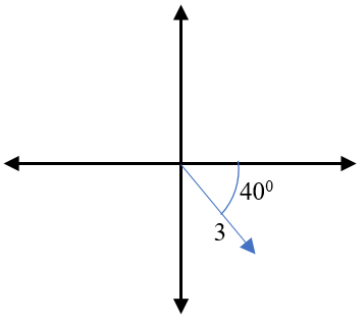
e *x*-coordinate is –2, *y*-coordinate is –4



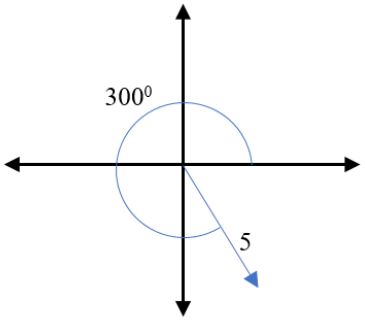
f *x*-coordinate is –5, *y*-coordinate is 2



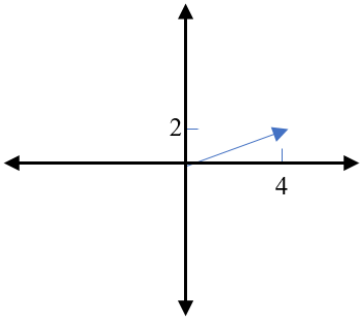
g Magnitude is 3, angle is 40° clockwise from x-axis.



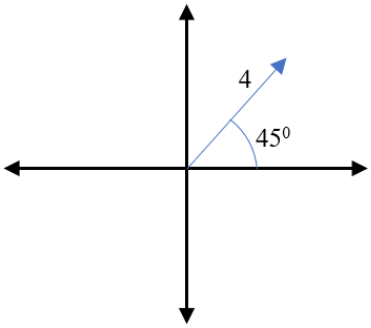
h Magnitude is 5, angle is 300° anticlockwise from the x-axis.



i x-coordinate is 4, y-coordinate is 2



j Magnitude is 4, angle is 45° anticlockwise from the x-axis.



Question 3

For each, use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where $(x_1, y_1) = (0, 0)$ are the given co-ordinates.

a $d = \sqrt{(2-0)^2 + (4-0)^2}$
$= \sqrt{4+16}$
$= \sqrt{20}$
$= 4.47$

b $d = \sqrt{(-1-0)^2 + (3-0)^2}$
$= \sqrt{1+9}$
$= \sqrt{10}$
$= 3.16$

c $d = \sqrt{(4-0)^2 + (-7-0)^2}$
$= \sqrt{16+49}$
$= \sqrt{65}$
$= 8.06$

d $d = \sqrt{(6-0)^2 + (1-0)^2}$
$= \sqrt{36+1}$
$= \sqrt{37}$
$= 6.08$

e $d = \sqrt{(-8-0)^2 + (3-0)^2}$
$= \sqrt{64+9}$
$= \sqrt{73}$
$= 8.54$

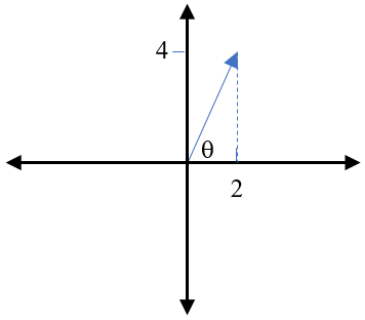
f $d = \sqrt{(5-0)^2 + (-2-0)^2}$
$= \sqrt{25+4}$
$= \sqrt{29}$
$= 5.39$

g $d = \sqrt{(-4-0)^2 + (-6-0)^2}$
$= \sqrt{16+36}$
$= \sqrt{52}$
$= 7.21$

h $d = \sqrt{(-3-0)^2 + (12-0)^2}$
$= \sqrt{9+144}$
$= \sqrt{153}$
$= 12.37$

Question 4

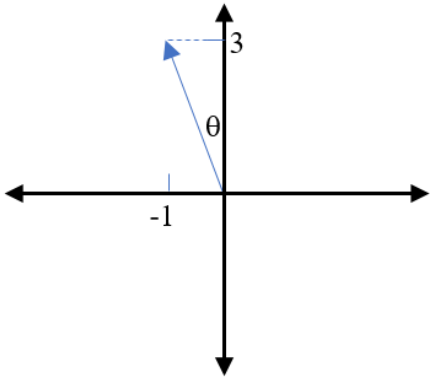
a



$$\tan(\theta) = \frac{4}{2} = 2$$

$$\theta = \tan^{-1}(2) = 63.4°$$

b

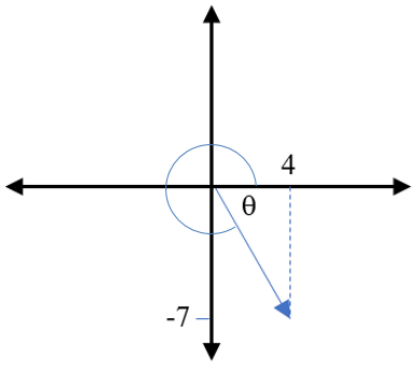


$$\tan(\theta) = \frac{1}{3}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right) = 18.4°$$

Required angle is $90° + 18.4° = 108.4°$

c

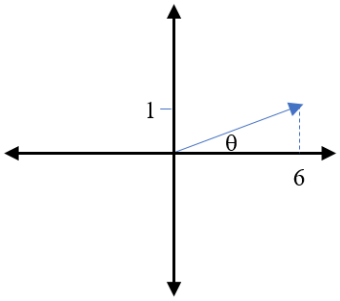


$$\tan(\theta) = \frac{7}{4} = 1.75$$

$$\theta = \tan^{-1}(1.75) = 60.3°$$

Required angle is $360° - 60.3° = 299.7°$

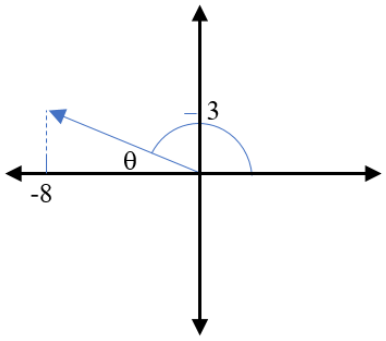
d



$$\tan(\theta) = \frac{1}{6}$$

$$\theta = \tan^{-1}\left(\frac{1}{6}\right) = 9.5°$$

e

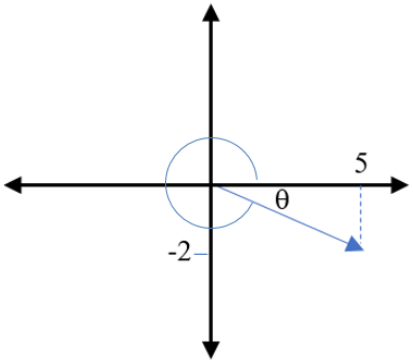


$$\tan(\theta) = \frac{3}{8}$$

$$\theta = \tan^{-1}\left(\frac{3}{8}\right) = 20.6°$$

Required angle is $180° - 20.6° = 159.4°$

f

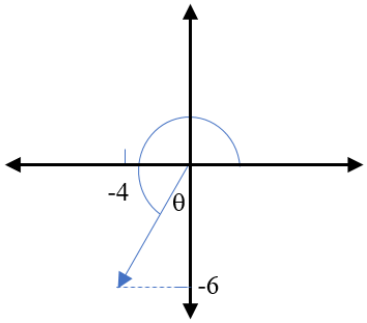


$$\tan(\theta) = \frac{2}{5} = 0.4$$

$$\theta = \tan^{-1}(0.4) = 21.8°$$

Required angle is $360° - 21.8° = 338.2°$

g

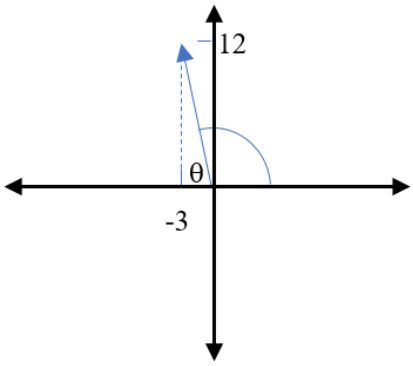


$$\tan(\theta) = \frac{4}{6}$$

$$\theta = \tan^{-1}\left(\frac{4}{6}\right) = 33.7°$$

Required angle is $270° - 33.7° = 236.3°$

h



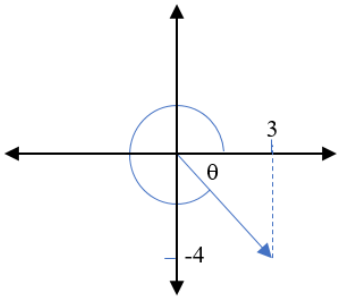
$$\tan(\theta) = \frac{12}{3} = 4$$

$$\theta = \tan^{-1}(4) = 76.0°$$

Required angle is $180° - 76.0° = 104.0°$

Question 5

a (5, 306.9°)



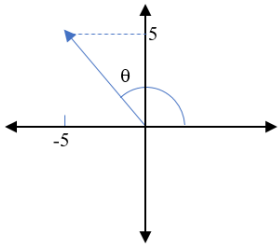
magnitude

$$\sqrt{(3-0)^2+(-4-0)^2} = \sqrt{9+16}$$
$$= 5$$

angle

$$\tan^{-1}\left(\frac{4}{3}\right) = 53.13°, \text{ required angle } 360° - 53.13° = 306.9°$$

b (7.07, 135°)



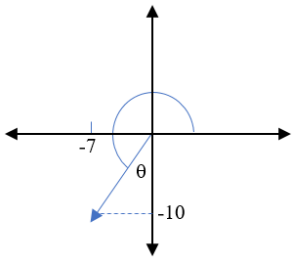
magnitude

$$\sqrt{(-5-0)^2+(5-0)^2} = \sqrt{50}$$
$$= 7.07$$

angle

$$\tan^{-1}\left(\frac{5}{5}\right) = 45°, \text{ required angle } 90° + 45° = 135°$$

c (12.21, 235.0°)



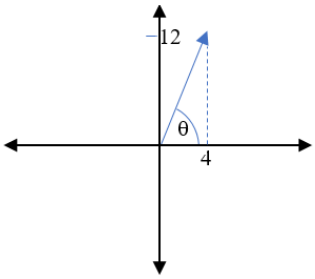
magnitude

$$\sqrt{(-7-0)^2+(-10-0)^2} = \sqrt{149}$$
$$= 12.21$$

angle

$$\tan^{-1}\left(\frac{7}{10}\right) = 35.0°\text{, required angle } 270° - 35.0° = 235.0°$$

d (12.65, 71.6°)



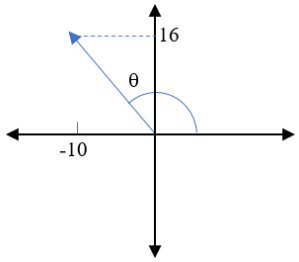
magnitude

$$\sqrt{(4-0)^2+(12-0)^2} = \sqrt{160}$$
$$= 12.65$$

angle

$$\tan^{-1}\left(\frac{12}{4}\right) = 71.6°\text{, required angle } 71.6°$$

e (18.87, 122.0°)



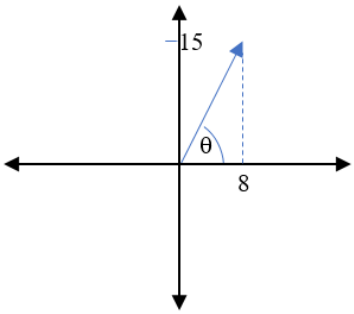
magnitude

$$\sqrt{(-10-0)^2+(16-0)^2} = \sqrt{356}$$
$$= 18.87$$

angle

$$\tan^{-1}\left(\frac{10}{16}\right) = 32.0°\text{, required angle } 90° + 32.0° = 122°$$

f (17, 61.9°)



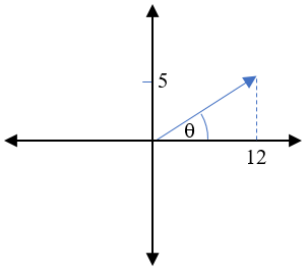
magnitude

$$\sqrt{(8-0)^2+(15-0)^2} = \sqrt{289}$$
$$= 17$$

angle

$$\tan^{-1}\left(\frac{15}{8}\right) = 61.9°\text{, required angle } 61.9°$$

g (13, 22.6°)



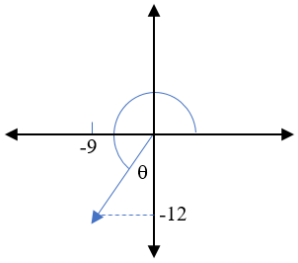
magnitude

$$\sqrt{(12-0)^2+(5-0)^2} = \sqrt{169}$$
$$= 13$$

angle

$$\tan^{-1}\left(\frac{5}{12}\right) = 22.6°, \text{ required angle } 22.6°$$

h (15, 233.1°)



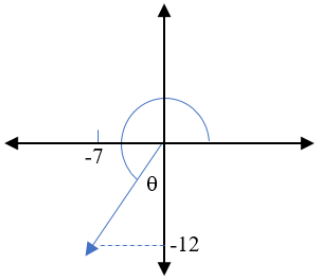
magnitude

$$\sqrt{(-9-0)^2+(-12-0)^2} = \sqrt{225}$$
$$= 15$$

angle

$$\tan^{-1}\left(\frac{9}{12}\right) = 36.9°, \text{ required angle } 270° - 36.9° = 233.1°$$

i (13.89, 239.7°)



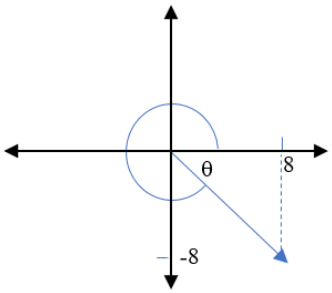
magnitude

$$\sqrt{(-7-0)^2+(-12-0)^2} = \sqrt{193}$$
$$= 13.89$$

angle

$$\tan^{-1}\left(\frac{7}{12}\right) = 30.3°, \text{ required angle } 270° - 30.3° = 239.7°$$

j (11.31, 315°)



magnitude

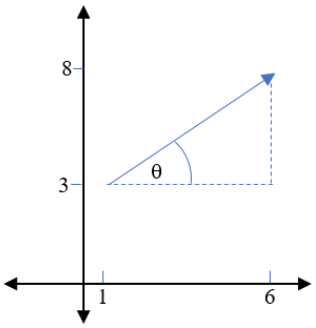
$$\sqrt{(8-0)^2+(-8-0)^2} = \sqrt{128}$$
$$= 11.31$$

angle

$$\tan^{-1}\left(\frac{8}{8}\right) = 45°, \text{ required angle } 360° - 45° = 315°$$

Question 6

a (7.07, 45°)



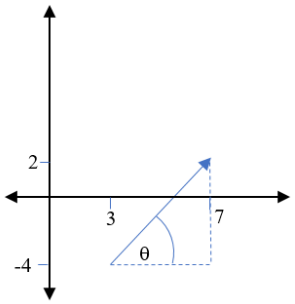
magnitude

$$\sqrt{(1-6)^2+(3-8)^2} = \sqrt{50}$$
$$= 7.07$$

angle

$$\tan^{-1}\left(\frac{5}{5}\right) = 45°, \text{ required angle } 45°$$

b (7.21, 56.3°)



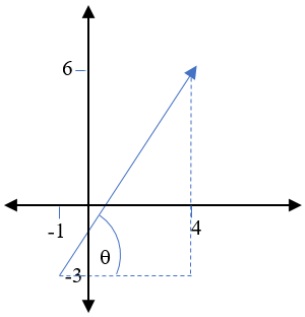
magnitude

$$\sqrt{(3-7)^2+(-4-2)^2} = \sqrt{52}$$
$$= 7.21$$

angle

$$\tan^{-1}\left(\frac{6}{4}\right) = 56.3°, \text{ required angle } 56.3°$$

c (10.30, 60.9°)



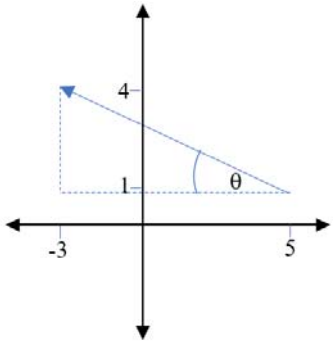
magnitude

$$\sqrt{(-1-4)^2+(-3-6)^2} = \sqrt{106}$$
$$= 10.30$$

angle

$$\tan^{-1}\left(\frac{9}{5}\right) = 60.9° \text{ required angle } 60.9°$$

d (8.54, 159.4°)



magnitude

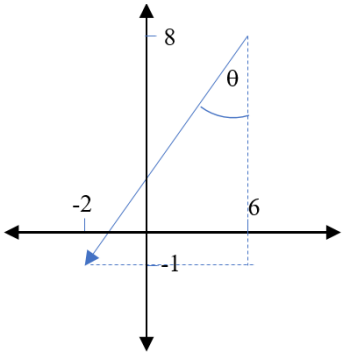
$$\sqrt{(5-(-3))^2+(1-4)^2} = \sqrt{73}$$
$$= 8.54$$

angle

$$\tan^{-1}\left(\frac{3}{8}\right) = 20.6°,$$

required angle 180° – 20.6° = 159.4°

e (12.04, 228.4°)



magnitude

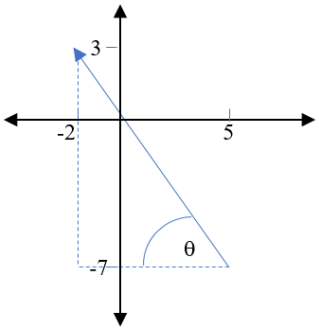
$$\sqrt{(6-(-2))^2+(8-(-1))^2} = \sqrt{145}$$
$$= 12.04$$

angle

$$\tan^{-1}\left(\frac{8}{9}\right) = 41.6°,$$

required angle 270° – 41.6° = 228.4°

f (12.21, 125.0°)



magnitude

$$\sqrt{(5-(-2))^2+(-7-(3))^2} = \sqrt{149}$$
$$= 12.21$$

angle

$$\tan^{-1}\left(\frac{10}{7}\right) = 55.0°,$$

required angle 180° – 55.0° = 125°

Question 7

Use a calculator set to degree mode.

a $(5\cos(30°), 5\sin(30°)) = (4.33, 2.50)$

b $(10\cos(300°), 10\sin(300°)) = (5, -8.66)$

c $(24\cos(90°), 24\sin(90°)) = (0, 24)$

d $(16\cos(135°), 16\sin(135°)) = (-11.31, 11.31)$

e $(28\cos(-120°), 28\sin(-120°)) = (-14, -24.25)$

f $(70\cos(270°), 70\sin(270°)) = (0, -70)$

g $(35\cos(0°), 35\sin(0°)) = (35, 0)$

h $(22\cos(180°), 22\sin(180°)) = (-22, 0)$

i $(4\cos(200°), 4\sin(200°)) = (-3.76, -1.37)$

j $(8\cos(-60°), 8\sin(-60°)) = (4, -6.93)$

Question 8

a $(4-1, 6-1) = (3, 5)$, $(5.83,\ 59.0°)$

$(4-1, 6-1) = (3, 5)$

magnitude: $\sqrt{(1-4)^2 + (1-6)^2} = 5.83$

angle: $\tan^{-1}\left(\frac{5}{3}\right) = 59.0°$

b $(10, 1)$, $(10.05,\ 5.7°)$

$(4-(-6), 6-5) = (10, 1)$

magnitude: $\sqrt{(-6-4)^2 + (5-6)^2} = 10.05$

angle: $\tan^{-1}\left(\frac{1}{10}\right) = 5.7°$

c $(8, 1)$, $(8.06, 7.1°)$

$(3-(-5), -4-(-5)) = (8, 1)$

magnitude: $\sqrt{(-5-3)^2+(-5-(-4))^2} = 8.06$

angle: $\tan^{-1}\left(\frac{1}{8}\right) = 7.1°$

d $(-6, -6)$, $(8.49, 225°)$

$(-5-1, -5-1) = (-6, -6)$

magnitude: $\sqrt{(-5-1)^2+(-5-1)^2} = 8.49$

angle: $\tan^{-1}\left(\frac{6}{6}\right) = 45°$, required angle: $180° + 45° = 225°$

e $(-9, -9)$, $(12.73, 135°)$

$(-6-3, 5-(-4)) = (-9, 9)$

magnitude: $\sqrt{(-6-3)^2+(5-(-4))^2} = 12.73$

angle: $\tan^{-1}\left(\frac{9}{9}\right) = 45°$, required angle: $180° - 45° = 135°$

f $(9,11)$, $(14.21, 50.7°)$

$(4-(-5), 6-(-5)) = (9,11)$

magnitude: $\sqrt{(4-(-5))^2+(6-(-5))^2} = 14.21$

angle: $\tan^{-1}\left(\frac{11}{9}\right) = 50.7°$

g $(-6, -5)$, $(7.81, 219.8°)$

$(-2-4, 1-6) = (-6, -5)$

magnitude: $\sqrt{(-2-4)^2+(1-6)^2} = 7.81$

angle: $\tan^{-1}\left(\frac{5}{6}\right) = 39.8°$, required angle: $180° + 39.8° = 219.8°$

Question 9

$a = 7, b = 4$, $(7.81, 219.8°)$

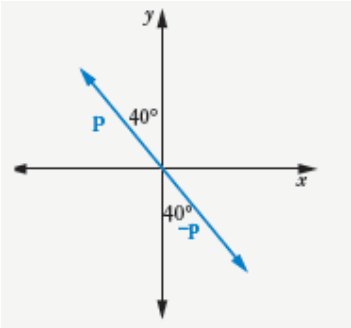
In component form, $\underset{\sim}{r} = (4-(-3), 2-(-2)) = (7, 4)$

Question 10

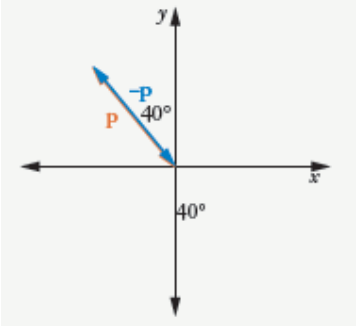
a The magnitude remains the same. The vector is rotated 180°.

The vector is of magnitude 6 in the direction S 40° E.

b



c



d **p** and –**p** occupy the same position, so the displacement vector is zero.

Exercise 1.03 Scalar multiples of vectors

Question 1

b = **a** (both have the same magnitude and direction)

c = –**a** (both have the same magnitude but opposite direction)

d = 2**a** (**d** has twice the length of **a** and both have the same direction)

e = –2**a** (**e** has twice the length of **a** and is in the opposite direction to **a**)

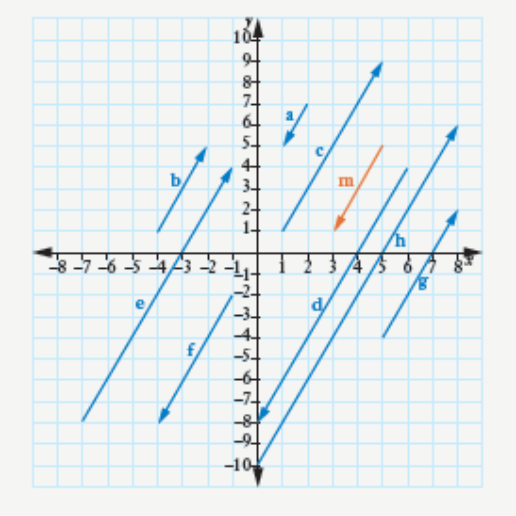
f = –3**a** (**f** is three times the length of **a** and is in the opposite direction to **a**)

$\mathbf{g} = \frac{1}{2}\mathbf{a}$ (**g** is half the length of **a** and has the same direction as **a**)

h = 4**a** (**h** is four times the length of **a** and has the same direction as **a**)

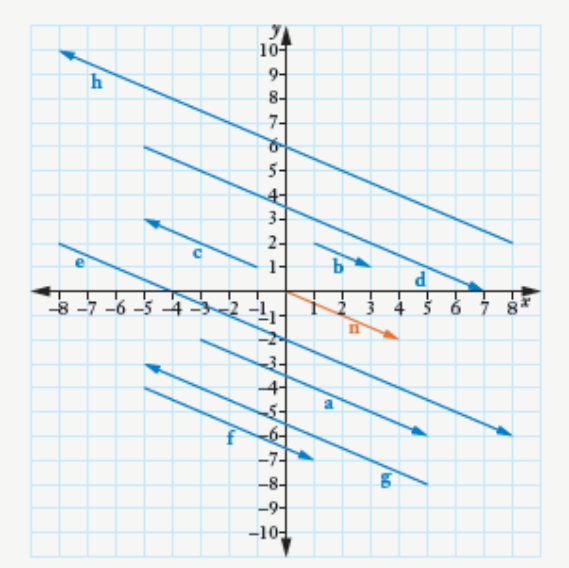
Question 2

The vectors have the same length and direction as shown here but their positions may vary.



Question 3

First draw vector **n** and use it to represent the given vectors, as shown below. The positions of your vectors may vary, but they must have the same length and direction.



Question 4

a $3\mathbf{p} = 3(8, 2) = (3 \times 8, 3 \times 2) = (24, 6)$

b $-\mathbf{p} = -(8, 2) = (-1 \times 8, -1 \times 2) = (-8, -2)$

c $\frac{1}{2}\mathbf{p} = \frac{1}{2}(8, 2) = (\frac{1}{2} \times 8, \frac{1}{2} \times 2) = (4, 1)$

d $\frac{3}{2}\mathbf{p} = \frac{3}{2}(8, 2) = (\frac{3}{2} \times 8, \frac{3}{2} \times 2) = (12, 3)$

e $-\frac{5}{2}\mathbf{p} = -\frac{5}{2}(8, 2) = (-\frac{5}{2} \times 8, -\frac{5}{2} \times 2) = (-20, -5)$

f $4\mathbf{p} = 4(8, 2) = (4 \times 8, 4 \times 2) = (32, 8)$

g $-\frac{1}{4}\mathbf{p} = -\frac{1}{4}(8, 2) = (-\frac{1}{4} \times 8, -\frac{1}{4} \times 2) = (-2, -0.5)$

h $\frac{3}{4}\mathbf{p} = \frac{3}{4}(8, 2) = (\frac{3}{4} \times 8, \frac{3}{4} \times 2) = (6, 1.5)$

Question 5

a $2\mathbf{w} = 2\begin{bmatrix} -12 \\ 30 \end{bmatrix} = \begin{bmatrix} -24 \\ 60 \end{bmatrix}$

b $-3\mathbf{w} = -3\begin{bmatrix} -12 \\ 30 \end{bmatrix} = \begin{bmatrix} 36 \\ -90 \end{bmatrix}$

c $\frac{1}{3}\mathbf{w} = \frac{1}{3}\begin{bmatrix} -12 \\ 30 \end{bmatrix} = \begin{bmatrix} -4 \\ 10 \end{bmatrix}$

d $-2.5\mathbf{w} = -2.5\begin{bmatrix} -12 \\ 30 \end{bmatrix} = \begin{bmatrix} 30 \\ -75 \end{bmatrix}$

e $-\frac{2}{3}\mathbf{w} = -\frac{2}{3}\begin{bmatrix} -12 \\ 30 \end{bmatrix} = \begin{bmatrix} 8 \\ -20 \end{bmatrix}$

f $1.5\mathbf{w} = 1.5\begin{bmatrix} -12 \\ 30 \end{bmatrix} = \begin{bmatrix} -18 \\ 45 \end{bmatrix}$

g $-\frac{1}{6}\mathbf{w} = -\frac{1}{6}\begin{bmatrix} -12 \\ 30 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$

h $0.5\mathbf{w} = 0.5\begin{bmatrix} -12 \\ 30 \end{bmatrix} = \begin{bmatrix} -6 \\ 15 \end{bmatrix}$

Question 6

a $2\mathbf{u} = (2 \times 5, 58°) = (10,\ 58°)$

b $-\mathbf{u} = (1 \times 5, 180° + 58°) = (5,\ 238°)$, direction is reversed

c $3\mathbf{u} = (3 \times 5, 58°) = (15,\ 58°)$

d $\frac{1}{2}\mathbf{u} = (\frac{1}{2} \times 5, 58°) = (2.5,\ 58°)$

e $-1.5\mathbf{u} = (1.5 \times 5, 180° + 58°) = (7.5,\ 238°)$, direction is reversed

f $2.5\mathbf{u} = (2.5 \times 5, 58°) = (12.5,\ 58°)$

g $-\frac{1}{5}\mathbf{u} = (\frac{1}{5} \times 5,\ 180° + 58°) = (1,\ 238°)$, direction is reversed

h $-6\mathbf{u} = (6 \times 5,\ 180° + 58°) = (30,\ 238°)$, direction is reversed

Question 7

a $4\mathbf{q} = (4 \times (-6), 4 \times 4) = (-24, 16)$

b $-2\mathbf{r} = (-2 \times 4, -2 \times (-10)) = (-8, 20)$

c $\frac{1}{3}\mathbf{p} = (\frac{1}{3} \times 9,\ \frac{1}{3} \times 3) = (3, 1)$

d $-2.5\mathbf{r} = (-2.5 \times 4, -2.5 \times (-10)) = (-10, 25)$

e $-\frac{3}{2}\mathbf{q} = (-\frac{3}{2} \times (-6), -\frac{3}{2} \times 4) = (9, -6)$

f $3\mathbf{p} = (3 \times 9, 3 \times 3) = (27, 9)$

g $\frac{3}{4}\mathbf{r} = (\frac{3}{4} \times 4, \frac{3}{4} \times (-10)) = (3, -7.5)$

h $-\frac{1}{4}\mathbf{q} = (-\frac{1}{4} \times (-6), -\frac{1}{4} \times 4) = (1.5, -1)$

Question 8

a $3\mathbf{d} = (3 \times 6, 36°) = (18, 36°)$

b $-\mathbf{f} = (1 \times 9, 218° - 180°) = (9, 38°)$, direction is reversed

c $2\mathbf{e} = (2 \times 4, 124°) = (8, 124°)$

d $-\frac{1}{3}\mathbf{d} = (\frac{1}{3} \times 6, 180° + 36°) = (2, 216°)$, direction is reversed

e $-2.5\mathbf{e} = (2.5 \times 4, 180° + 124°) = (10, 304°)$, direction is reversed

f $1.5\mathbf{d} = (1.5 \times 6, 36°) = (9, 36°)$

g $-\frac{1}{3}\mathbf{f} = (\frac{1}{3} \times 9, 218° - 180°) = (3, 38°)$, direction is reversed

h $-5\mathbf{e} = (5 \times 4, 180° + 124°) = (20, 304°)$, direction is reversed

Exercise 1.04 Addition of vectors

Question 1

Take negative to be west and positive to be east.

a $9+4=13$, 13 right

b $3-16+8=-5$, 5 left

c $-12+17=5$, 5 right

d $15-15=0$

e $-13+6-10=-17$, 17 left

Question 2

Take negative to be down and positive to be up

a $50-90=-40$, 40 N down

b $78-27=51$, 51 N up

c $32-56-48=-72$, 72 N down

d $22+37-33=26$, 26 N up

e $46-46=0$
Resultant force vertically is zero.
Horizontal force is 38 N left.

Question 3

a (3, 0°)

$-2+5=3$, 3 units on positive x-axis, magnitude 3, angle is 0° from positive x-axis.

b (2, 270°)

$5-7=-2$, 2 units on negative y-axis, magnitude 2,

angle is 270° from positive x-axis.

c (2, 180°)

$-7+5=-2$, 2 units on negative x-axis, magnitude 2,

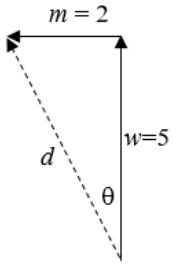
angle is 180° from positive x-axis.

d (12, 90°)

$5+7=12$, 12 units on positive y-axis, magnitude 12,

angle is 90° from positive x-axis.

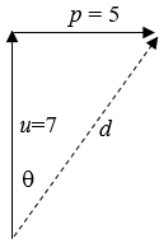
e (5.4, 111.8°)



$d=\sqrt{2^2+5^2}=5.4$

$\theta=\tan^{-1}\left(\frac{2}{5}\right)=21.8°$, required angle is 90° + 21.8° = 111.8°

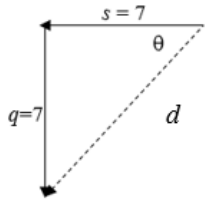
f (8.6, 54.5°)



$d = \sqrt{7^2 + 5^2} = 8.6$

$\theta = \tan^{-1}\left(\frac{5}{7}\right) = 35.5°$, required angle is 90° – 35.5° = 54.5°

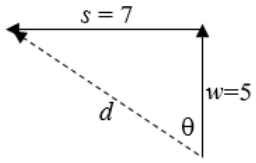
g (9.9, 225°)



$d = \sqrt{7^2 + 7^2} = 9.9$

$\theta = \tan^{-1}\left(\frac{7}{7}\right) = 45°$, required angle is 180° + 45° = 225°

h (8.6, 144.5°)

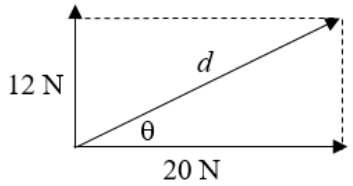


$d = \sqrt{5^2 + 7^2} = 8.6$

$\theta = \tan^{-1}\left(\frac{7}{5}\right) = 54.5°$, required angle is 90° + 54.5° = 144.5°

Question 4

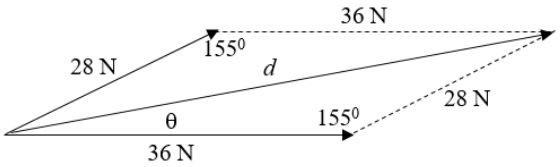
a (23.3, 31.0°)



$d = \sqrt{12^2 + 20^2} = 23.3$

$\theta = \tan^{-1}\left(\frac{12}{20}\right) = 31.0°$

b (62.5, 10.9°)



Apply the cosine rule

$d^2 = 36^2 + 28^2 - 2 \times 36 \times 28 \times \cos(155°)$

$d = 62.5$

Apply the sine rule

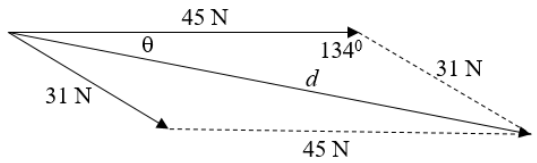
$$\frac{\sin(\theta)}{28} = \frac{\sin(155°)}{62.5}$$

$$\sin(\theta) = 28 \times \frac{\sin(155°)}{62.5}$$

$$= 0.189$$

$$\theta = \sin^{-1}(0.189) = 10.9°$$

c (70.2, 341.5°)



Apply the cosine rule

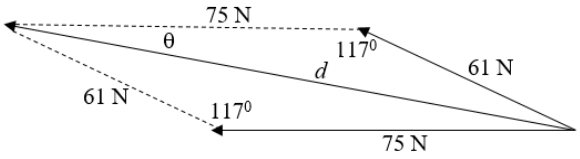
$$d^2 = 31^2 + 45^2 - 2 \times 31 \times 45 \times \cos(134°)$$
$$d = 70.2$$

Apply the sine rule

$$\frac{\sin(\theta)}{31} = \frac{\sin(134°)}{70.2}$$
$$\sin(\theta) = 31 \times \frac{\sin(134°)}{70.2}$$
$$= 0.318$$
$$\theta = \sin^{-1}(0.318) = 18.5°$$

Required angle 360° – 18.5° = 341.5°

d (116.2, 152.1°)



Apply the cosine rule

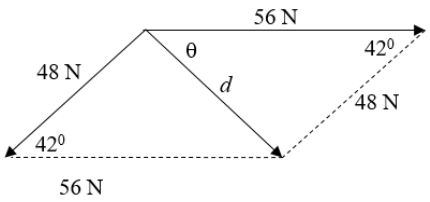
$$d^2 = 61^2 + 75^2 - 2 \times 61 \times 75 \times \cos(117°)$$
$$d = 116.2$$

Apply the sine rule

$$\frac{\sin(\theta)}{61} = \frac{\sin(117°)}{116.2}$$
$$\sin(\theta) = 61 \times \frac{\sin(117°)}{116.2}$$
$$= 0.4677$$
$$\theta = \sin^{-1}(0.4677) = 27.9°$$

Required angle 180° – 27.9° = 152.1°

e (38.0, 302.3°)



Apply the cosine rule

$d^2 = 48^2 + 56^2 - 2 \times 48 \times 56 \times \cos(42°)$

$d = 38.0$

Apply the sine rule

$\frac{\sin(\theta)}{48} = \frac{\sin(42°)}{38.0}$

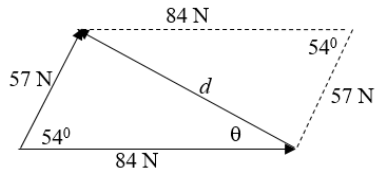
$\sin(\theta) = 48 \times \frac{\sin(42°)}{38.0}$

$= 0.8452$

$\theta = \sin^{-1}(0.8452) = 57.7°$

Required angle $360° - 57.7° = 302.3°$

f (68.4, 137.6°)



Apply the cosine rule

$d^2 = 57^2 + 84^2 - 2 \times 57 \times 84 \times \cos(54°)$

$d = 68.4$

Apply the sine rule

$\frac{\sin(\theta)}{57} = \frac{\sin(54°)}{68.4}$

$\sin(\theta) = 57 \times \frac{\sin(54°)}{68.4}$

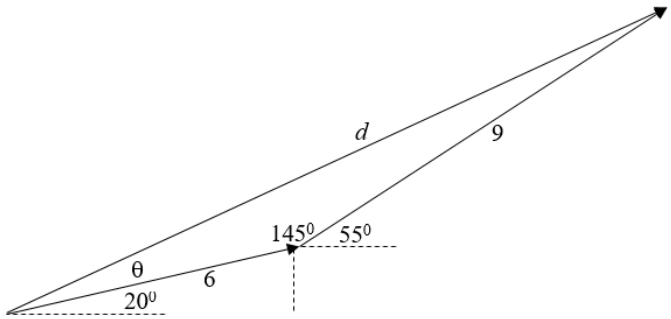
$= 0.6741$

$\theta = \sin^{-1}(0.6741) = 42.4°$

Required angle $180° - 42.4° = 137.6°$

Question 5

$(14.3, 41.2°)$



Apply the cosine rule

$$d^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos(145°)$$
$$d = 14.3$$

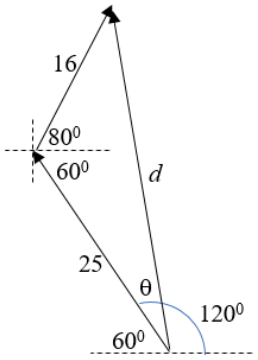
Apply the sine rule

$$\frac{\sin(\theta)}{9} = \frac{\sin(145°)}{14.3}$$
$$\sin(\theta) = 9 \times \frac{\sin(145°)}{14.3}$$
$$= 0.3610$$
$$\theta = \sin^{-1}(0.3610) = 21.2°$$

Required angle 21.2° + 20° = 41.2°

Question 6

(38.7, 104.6°)



Apply the cosine rule

$$d^2 = 16^2 + 25^2 - 2 \times 16 \times 25 \times \cos(140°)$$
$$d = 38.7$$

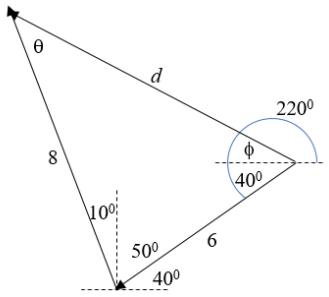
Apply the sine rule

$$\frac{\sin(\theta)}{16} = \frac{\sin(140°)}{38.7}$$
$$\sin(\theta) = 16 \times \frac{\sin(140°)}{38.7}$$
$$= 0.2658$$
$$\theta = \sin^{-1}(0.2658) = 15.4°$$

Required angle 120° – 15.4° = 104.6°

Question 7

a (7.21, 146.1°)



Apply the cosine rule

$d^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos(60°)$

$d = 7.21$

Apply the sine rule

$$\frac{\sin(\theta)}{6} = \frac{\sin(60°)}{7.21}$$

$$\sin(\theta) = 6 \times \frac{\sin(60°)}{7.21}$$

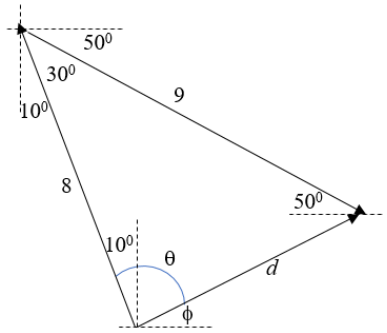
$$= 0.7207$$

$\theta = \sin^{-1}(0.7207) = 46.1°$

$\varphi = 180° - 60° - 40° - \theta = 33.9°$

Required angle $180° - \varphi = 146.1°$

b (4.50, 12.62°)



Apply the cosine rule

Note: Take the value of d to at least 4 decimal places for improved accuracy in the answer.

$d^2 = 9^2 + 8^2 - 2 \times 9 \times 8 \times \cos(30°)$

$d \approx 4.5047$

Apply the sine rule

$$\frac{\sin(\theta)}{9} = \frac{\sin(30°)}{4.50}$$

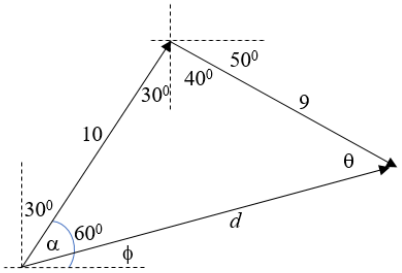
$$\sin(\theta) = 9 \times \frac{\sin(30°)}{4.5047}$$

$$= 0.998956$$

$$\theta = \sin^{-1}(0.998956) = 87.38°$$

Required angle $\varphi = 90° - (\theta - 10°) = 12.62°$

c (10.93, 9.3°)



Apply the cosine rule

$$d^2 = 9^2 + 10^2 - 2 \times 9 \times 10 \times \cos(70°)$$
$$d \approx 10.93$$

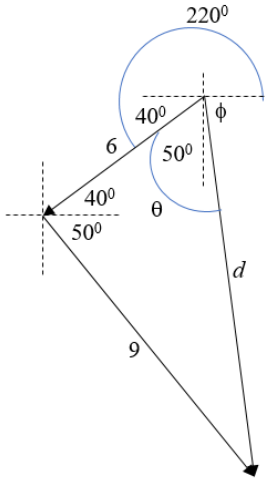
Apply the sine rule

$$\frac{\sin(\theta)}{10} = \frac{\sin(70°)}{10.93}$$
$$\sin(\theta) = 10 \times \frac{\sin(70°)}{10.93}$$
$$= 0.86$$
$$\theta = \sin^{-1}(0.86) = 59.3°$$

$$\alpha = 180° - \theta - 70° = 50.7°$$

Required angle $60° - \varphi = 9.3°$

d (10.82, 276.3°)



Apply Pythagoras' theorem.

$d^2 = 9^2 + 6^2$

$d \approx 10.82$

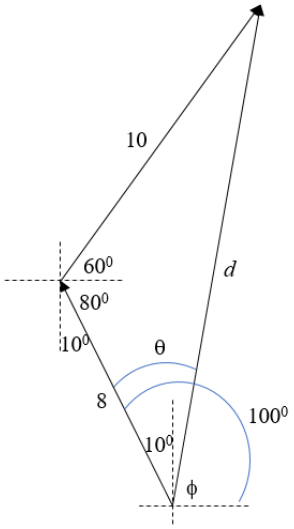
$\tan(\theta) = \frac{9}{6}$

$\theta = 56.3°$

$\varphi = 90° - (\theta - 50°) = 83.7°$

Required angle $360° - \varphi = 276.3°$

e (16.93, 77.7°)



Apply the cosine rule

$d^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos(140°)$

$d \approx 16.93$

Apply the sine rule

$\frac{\sin(\theta)}{10} = \frac{\sin(140°)}{16.93}$

$\sin(\theta) = 10 \times \frac{\sin(140°)}{16.93}$

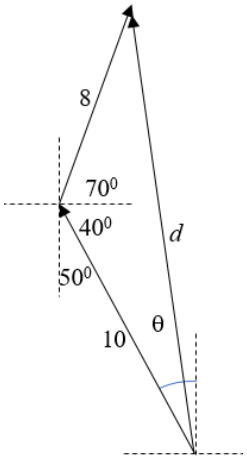
$= 0.38$

$\theta = \sin^{-1}(0.38) = 22.3°$

$\varphi = 100° - \theta = 77.7°$

Question 8

a (14.79, 109.4°)



Apply the cosine rule

$d^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \times \cos(110°)$

$d = 14.79$

Apply the sine rule

$$\frac{\sin(\theta)}{8} = \frac{\sin(110°)}{14.79}$$

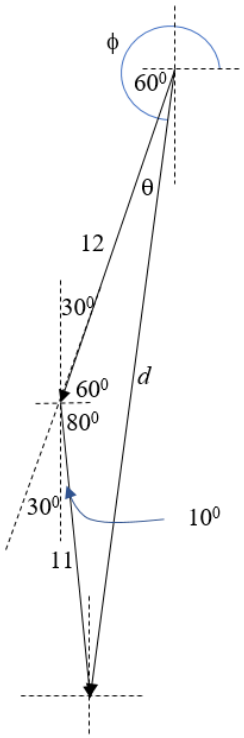
$$\sin(\theta) = 8 \times \frac{\sin(110°)}{14.79}$$

$$= 0.508$$

$\theta = \sin^{-1}(0.508) = 30.5°$

Required angle $90° + 50° - \theta = 109.4°$

b (21.62, 259.1°)



Apply the cosine rule

$d^2 = 11^2 + 12^2 - 2 \times 11 \times 12 \times \cos(140°)$

$d = 21.62$

Apply the sine rule

$$\frac{\sin(\theta)}{11} = \frac{\sin(140°)}{21.62}$$

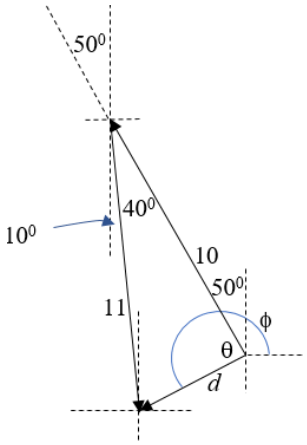
$$\sin(\theta) = 11 \times \frac{\sin(140°)}{21.62}$$

$$= 0.327$$

$\theta = \sin^{-1}(0.327) = 19.1°$

Required angle $180° + 60° + \theta = 259.1°$

c (7.24, 217.6°)



Apply the cosine rule

$d^2 = 11^2 + 10^2 - 2 \times 11 \times 10 \times \cos(40°)$

$d = 7.24$

Apply the sine rule

$$\frac{\sin(\theta)}{11} = \frac{\sin(40°)}{7.24}$$

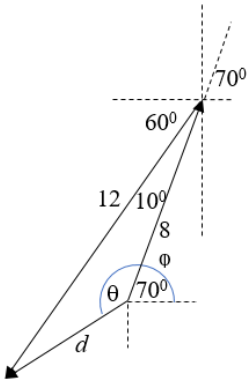
$$\sin(\theta) = 11 \times \frac{\sin(40°)}{7.24}$$

$$= 0.977$$

$\theta = \sin^{-1}(0.327) = 77.6°$

Required angle $\varphi = 90° + 50° + \theta = 217.6°$

d (4.35, 221.4°)



Apply the cosine rule

$d^2 = 12^2 + 8^2 - 2 \times 12 \times 8 \times \cos(10°)$

$d = 4.35$

Apply the sine rule

$$\frac{\sin(\theta)}{12} = \frac{\sin(10°)}{4.35}$$

$$\sin(\theta) = 12 \times \frac{\sin(10°)}{4.35}$$

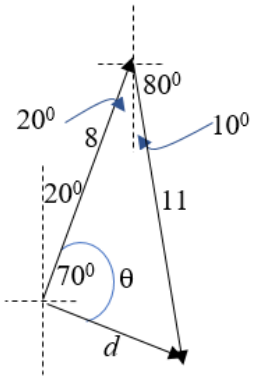
$$= 0.479$$

$\theta = \sin^{-1}(0.479) = 28.6°$

Since $\theta > 90°$, take $\theta = 180° - 28.6° = 151.4°$

Required angle $\varphi = 70° + \theta = 221.4°$

e (5.71, 355.6°)



Apply the cosine rule

$$d^2 = 11^2 + 8^2 - 2 \times 11 \times 8 \times \cos(30°)$$
$$d = 5.71$$

Apply the sine rule

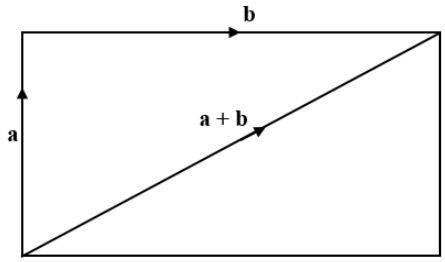
$$\frac{\sin(\theta)}{11} = \frac{\sin(30°)}{5.71}$$
$$\sin(\theta) = 11 \times \frac{\sin(30°)}{5.71}$$
$$= 0.963$$
$$\theta = \sin^{-1}(0.963) = 74.4°$$

Required angle

$$90° - (\theta + 20°) = 90° - 94.4°$$
$$= -4.4°$$
$$= 360° - 4.4°$$
$$= 355.6°$$

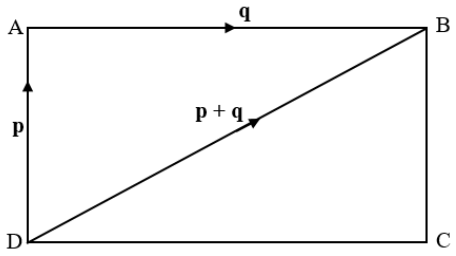
Question 9

a + **b**



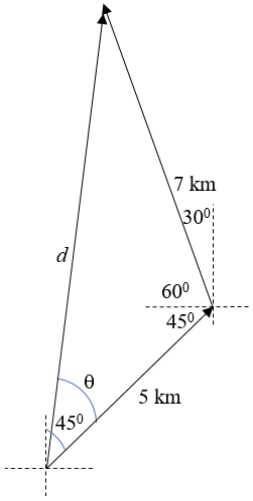
Question 10

p + **q** is the length of the diagonal of the rectangle.



Question 11

(9.60, 89.8°)



Apply the cosine rule

$$d^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos(105°)$$
$$d = 9.60$$

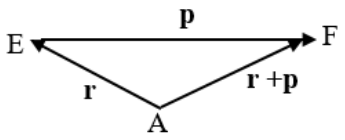
Apply the sine rule

$$\frac{\sin(\theta)}{7} = \frac{\sin(105°)}{9.60}$$
$$\sin(\theta) = 7 \times \frac{\sin(105°)}{9.60}$$
$$= 0.704$$
$$\theta = \sin^{-1}(0.963) = 44.8°$$

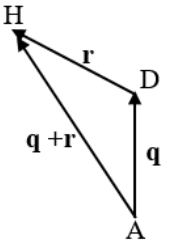
Required angle $45° + \theta = 89.8°$

Question 12

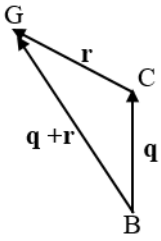
a $\mathbf{r} + \mathbf{p}$



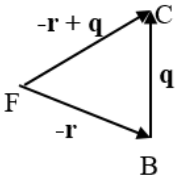
b $\mathbf{q} + \mathbf{r}$



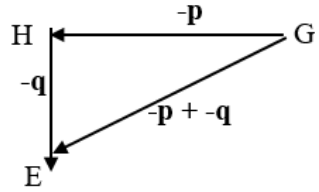
c $\mathbf{q} + \mathbf{r}$



d $-\mathbf{r} + \mathbf{q}$



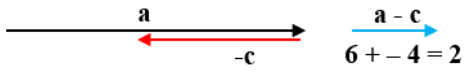
e $-\mathbf{p} + -\mathbf{q}$



Question 1

a (2, 0°)

$\mathbf{a} + -\mathbf{c} = 6 + (-4) = 2$, angle 0°

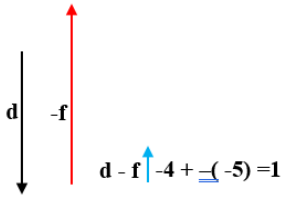


b (6, 0°)

$\mathbf{c} + -\mathbf{e} = 4 + [-(-2)] = 6$, angle from positive x-axis 0°

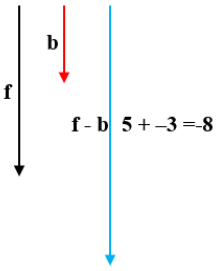
c (1, 90°)

$\mathbf{d} + (-\mathbf{f}) = -4 + [-(-5)] = 1$, angle from positive x-axis 90°



d (8, 270°)

$\mathbf{f} + (-\mathbf{b}) = -5 + (-3) = -8$, angle from positive x-axis 270°

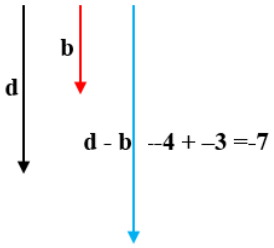


e (5, 180°)

$\mathbf{e} + (-\mathbf{g}) = -2 + (-3) = -5$, angle from positive x-axis 180°

f (7, 270°)

$\mathbf{d} + (-\mathbf{b}) = -4 + (-3) = -7$, angle from positive x-axis 270°



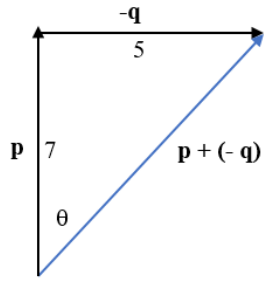
Question 2

a (8.6, 54.5°)

magnitude $|\mathbf{p} + (-\mathbf{q})| = \sqrt{7^2 + 5^2} = 8.6$

$\theta = \tan^{-1}\left(\frac{5}{7}\right) = 35.5°$

required angle $90° - \theta = 54.5°$

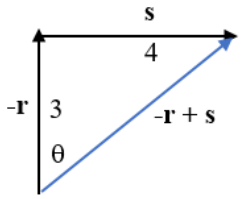


b (5, 36.9°)

magnitude $|-\mathbf{r} + \mathbf{s}| = \sqrt{3^2 + 4^2} = 5$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

required angle $90° - \theta = 36.9°$

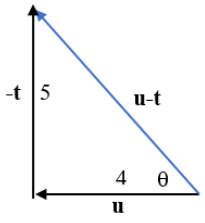


c (6.4, 128.7°)

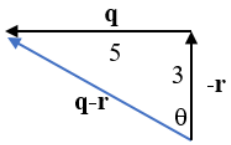
magnitude $|\mathbf{u} - \mathbf{t}| = \sqrt{4^2 + 5^2} = 6.4$

$\theta = \tan^{-1}\left(\frac{5}{4}\right) = 51.3°$

required angle $180° - \theta = 128.7°$



d (5.8, 149.0°)

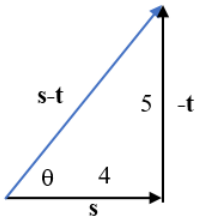


magnitude $|\mathbf{q}-\mathbf{r}| = \sqrt{3^2+5^2} = 5.8$

$\theta = \tan^{-1}\left(\frac{5}{3}\right) = 59.0°$

required angle $90° + \theta = 149.0°$

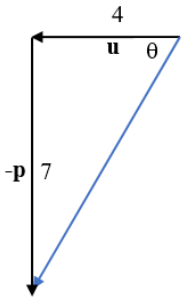
e (6.4, 51.3°)



magnitude $|\mathbf{s}-\mathbf{t}| = \sqrt{4^2+5^2} = 6.40$

$\theta = \tan^{-1}\left(\frac{5}{4}\right) = 51.3°$

f (8.1, 240.3°)



magnitude $|\mathbf{q}-\mathbf{r}| = \sqrt{4^2+7^2} = 8.1$

$\theta = \tan^{-1}\left(\frac{7}{4}\right) = 60.3°$

required angle $180° + \theta = 240.3°$

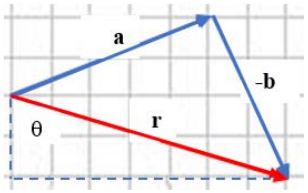
Question 3

a (7.3, 344.1°)

$|\mathbf{r}| = \sqrt{7^2 + 2^2} = 7.3$

$\theta = \tan^{-1}\left(\frac{7}{2}\right) = 74.1°$

required angle $270° + \theta = 344.1°$

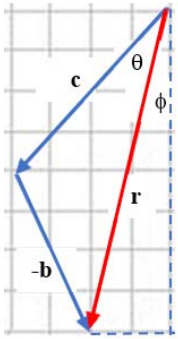


b (8.2, 256.0°)

$|\mathbf{r}| = \sqrt{8^2 + 2^2} = 8.2$

$\phi = \tan^{-1}\left(\frac{2}{8}\right) = 14.0°$

required angle $\theta = 270° - \phi = 256.0°$

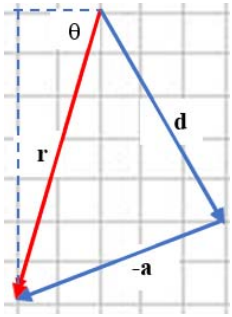


c (7.3, 254.1°)

$|\mathbf{r}| = \sqrt{7^2 + 2^2} = 7.3$

$\theta = \tan^{-1}\left(\frac{7}{2}\right) = 74.1°$

required angle $180° + \theta = 254.1°$

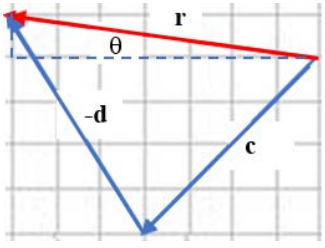


d (7.1, 171.9°)

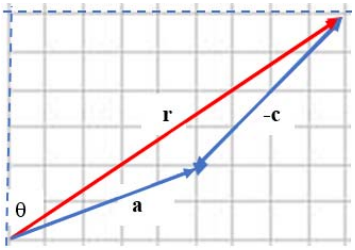
$|\mathbf{r}| = \sqrt{7^2 + 1^2} = 7.1$

$\theta = \tan^{-1}\left(\frac{1}{7}\right) = 8.1°$

required angle $180° - \theta = 171.9°$



e (10.8, 33.7°)

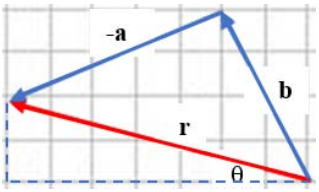


$|\mathbf{r}| = \sqrt{6^2 + 9^2} = 10.8$

$\theta = \tan^{-1}\left(\frac{9}{6}\right) = 56.3°$

required angle $90° - \theta = 33.7°$

f (7.3, 164.1°)



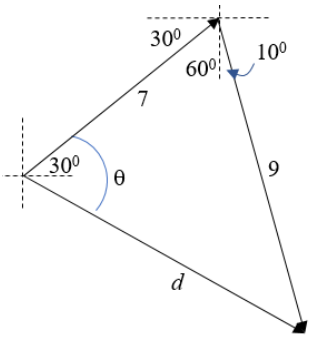
$|\mathbf{r}| = \sqrt{2^2 + 7^2} = 7.3$

$\theta = \tan^{-1}\left(\frac{2}{7}\right) = 15.9°$

required angle $180° - \theta = 164.1°$

Question 4

a (9.32, 324.9°)



Apply the cosine rule

$d^2 = 9^2 + 7^2 - 2 \times 9 \times 7 \times \cos(70°)$

$d = 9.32$

Apply the sine rule

$$\frac{\sin(\theta)}{9} = \frac{\sin(70°)}{9.32}$$

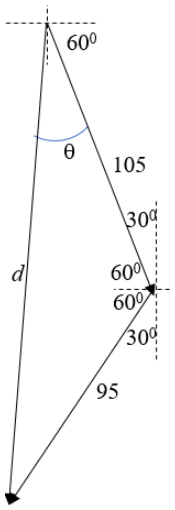
$$\sin(\theta) = 9 \times \frac{\sin(70°)}{9.32}$$

$$= 0.907$$

$\theta = \sin^{-1}(0.907) = 65.2°$

Required angle $360° - (\theta - 30°) = 324.9°$

b (173.28, 271.7°)



Apply the cosine rule

$d^2 = 95^2 + 105^2 - 2 \times 95 \times 105 \times \cos(120°)$

$d = 173.28$

Apply the sine rule

$$\frac{\sin(\theta)}{95} = \frac{\sin(120°)}{173.28}$$

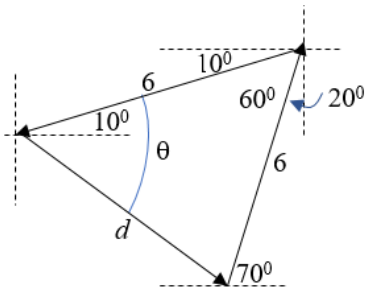
$$\sin(\theta) = 95 \times \frac{\sin(120°)}{173.28}$$

$$= 0.475$$

$\theta = \sin^{-1}(0.475) = 28.3°$

Required angle $360° - 60° - \theta = 271.7°$

c $(6,\ 310°)$



Apply the cosine rule

$d^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \times \cos(60°)$

$d = 6$

Apply the sine rule

$$\frac{\sin(\theta)}{6} = \frac{\sin(60°)}{6}$$

$$\sin(\theta) = 6 \times \frac{\sin(60°)}{6}$$

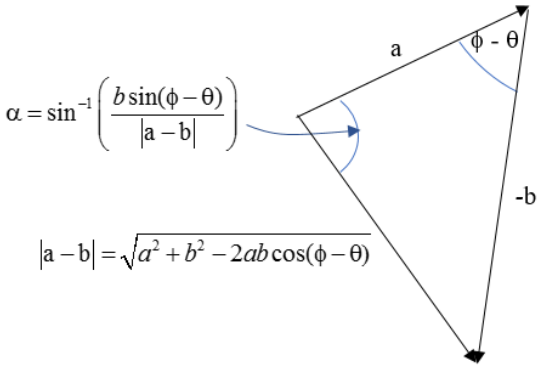
$$= 0.867$$

$\theta = \sin^{-1}(0.867) = 60°$

Required angle $360° - (\theta - 10°) = 310°$

Question 5

a



Angle of resultant vector $(360+\theta-\alpha)^0$

(15.77, 232.5°)

$a = 6 \quad b = 10 \quad \theta = 220° \quad \phi = 60°$

magnitude $\sqrt{6^2+10^2-2\times6\times10\times\cos(-160°)} = 15.77$

$$\alpha = \sin^{-1}\left[\frac{10\sin(-160°)}{15.77}\right] = -12.5° = 347.5°$$

angle of resultant vector $360+220-347.5 = 232.5°$

b (16.42, 295.9°)

$a = 9 \quad b = 8 \quad \theta = -50° \quad \phi = 100°$

magnitude $\sqrt{9^2+8^2-2\times9\times8\times\cos(150°)} = 16.42$

$$\alpha = \sin^{-1}\left(\frac{8\sin(150°)}{16.42}\right) = 14.1°$$

angle of resultant vector $360+(-50)-14.1 = 295.9°$

c (15.57, 92.9°)

$a = 10 \quad b = 9 \quad \theta = 60° \quad \phi = -50°$

magnitude $\sqrt{10^2 + 9^2 - 2 \times 10 \times 9 \times \cos(-110°)} = 15.57$

$\alpha = \sin^{-1}\left[\frac{9\sin(-110°)}{15.57}\right] = -32.9° = 327.1°$

angle of resultant vector $360 + 60 - 327.1 = 92.9°$

d (12.17, 74.7°)

$a = 8 \quad b = 6 \quad \theta = 100° \quad \phi = 220°$

magnitude $\sqrt{8^2 + 6^2 - 2 \times 8 \times 6 \times \cos(120°)} = 12.17$

$\alpha = \sin^{-1}\left[\frac{6\sin(120°)}{12.17}\right] = 25.3°$

angle of resultant vector $360 + 100 - 25.3 = 434.7°$

$434.7° - 360° = 74.7°$

e (12.17, 254.7°)

$a = 6 \quad b = 8 \quad \theta = 220° \quad \phi = 100°$

magnitude $\sqrt{6^2 + 8^2 - 2 \times 6 \times 8 \times \cos(-120°)} = 12.17$

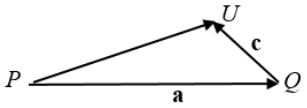
$\alpha = \sin^{-1}\left[\frac{8\sin(-120)}{12.17}\right] = -34.7° = 325.3°$

angle of resultant vector $360 + 220 - 325.3 = 254.7°$

Question 6

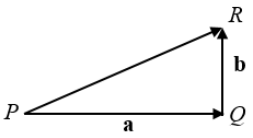
a $-\mathbf{a} - \mathbf{c}$

$\mathbf{PU} = \mathbf{a} + \mathbf{c}$, $\mathbf{UP} = -\mathbf{PU} = -\mathbf{a} - \mathbf{c}$



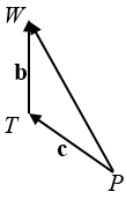
b $-\mathbf{a} - \mathbf{b}$

$\mathbf{PR} = \mathbf{a} + \mathbf{b}$, $\mathbf{RP} = -\mathbf{PR} = -\mathbf{a} - \mathbf{b}$



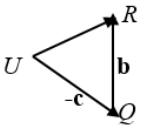
c $-\mathbf{b} - \mathbf{c}$

$\mathbf{PW} = \mathbf{c} + \mathbf{b}$, $\mathbf{WP} = -\mathbf{PW} = -\mathbf{b} - \mathbf{c}$



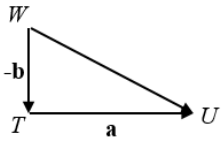
d $-\mathbf{c} + \mathbf{b}$

$\mathbf{UR} = -\mathbf{c} + \mathbf{b}$



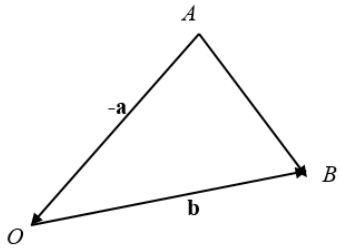
e $-\mathbf{b} + \mathbf{a}$

$\mathbf{WU} = -\mathbf{b} + \mathbf{a}$



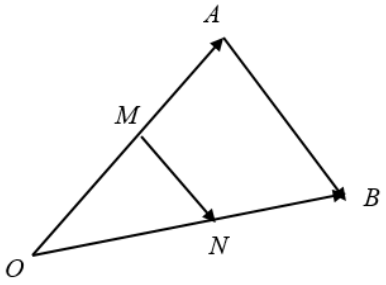
Question 7

a $\mathbf{AB} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$



b $\mathbf{OM} = \frac{1}{2}\mathbf{OA} = \frac{1}{2}\mathbf{a}$

$\mathbf{ON} = \frac{1}{2}\mathbf{OB} = \frac{1}{2}\mathbf{b}$

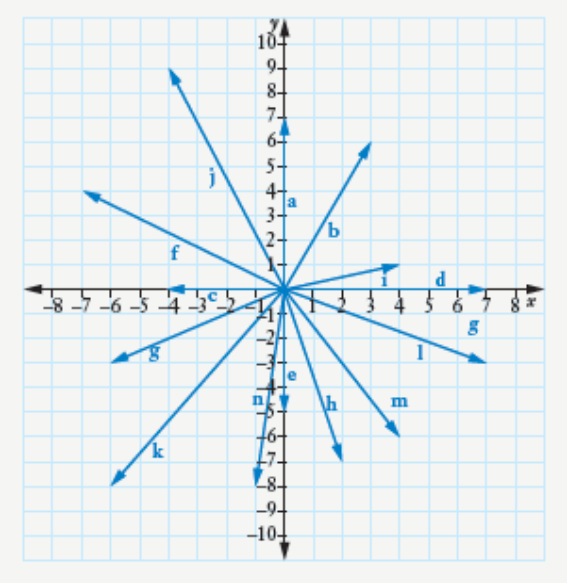


$\mathbf{OA} + \mathbf{AB} = \mathbf{OB} \Rightarrow \mathbf{AB} = \mathbf{OB} - \mathbf{OA} = \mathbf{b} - \mathbf{a}$

$\mathbf{OM} + \mathbf{MN} = \mathbf{ON} \Rightarrow \mathbf{MN} = \mathbf{ON} - \mathbf{OM} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} = \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\mathbf{AB}$

Exercise 1.06 Unit vectors

Question 1



Question 2

7 units

magnitude (length) of **m** is 1 unit, 7**m** has magnitude $7 \times 1 = 7$ units

Question 3

a $\left(\frac{3}{5}, \frac{4}{5}\right)$

$\mathbf{p} = (3,4)$, $|\mathbf{p}| = \sqrt{3^2 + 4^2} = 5$, $\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{5}(3,4) = \left(\frac{3}{5}, \frac{4}{5}\right)$

b $\left(-\frac{\sqrt{17}}{17}, \frac{4\sqrt{17}}{17}\right)$

$\mathbf{p} = (-2,8)$, $|\mathbf{p}| = \sqrt{(-2)^2 + 8^2} = \sqrt{68} = 2\sqrt{17}$,

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{2\sqrt{17}}(-2,8) = \left(-\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}}\right) = \left(-\frac{\sqrt{17}}{17}, \frac{4\sqrt{17}}{17}\right)$

c $\begin{bmatrix} \frac{5\sqrt{26}}{26} \\ -\frac{\sqrt{26}}{26} \end{bmatrix}$

$\mathbf{p} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$, $|\mathbf{p}| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$, $\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{\sqrt{26}}\begin{bmatrix} 5 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{5}{\sqrt{26}} \\ -\frac{1}{\sqrt{26}} \end{bmatrix} = \begin{bmatrix} \frac{5\sqrt{26}}{26} \\ -\frac{\sqrt{26}}{26} \end{bmatrix}$

d $\left(-\frac{2\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}\right)$

$\mathbf{p} = (-6,-3)$, $|\mathbf{p}| = \sqrt{(-6)^2 + (-3)^2} = \sqrt{45} = 3\sqrt{5}$,

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{3\sqrt{5}}(-6,-3) = \left(-\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right) = \left(-\frac{2\sqrt{5}}{5}, -\frac{\sqrt{5}}{5}\right)$

e $\left(-\frac{5\sqrt{34}}{34}, \frac{3\sqrt{34}}{34}\right)$

$\mathbf{p} = (-5,3)$, $|\mathbf{p}| = \sqrt{(-5)^2 + 3^2} = \sqrt{34}$,

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{\sqrt{34}}(-5,3) = \left(-\frac{5}{\sqrt{34}}, \frac{3}{\sqrt{34}}\right) = \left(-\frac{5\sqrt{34}}{34}, \frac{3\sqrt{34}}{34}\right)$

f $\begin{bmatrix} -\frac{\sqrt{10}}{10} \\ -\frac{3\sqrt{10}}{10} \end{bmatrix}$

$\mathbf{p} = \begin{bmatrix} -2 \\ -6 \end{bmatrix}$, $|\mathbf{p}| = \sqrt{(-2)^2 + (-6)^2} = \sqrt{40} = 2\sqrt{10}$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{2\sqrt{10}} \begin{bmatrix} -2 \\ -6 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{10}} \\ -\frac{3}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{10}}{10} \\ -\frac{3\sqrt{10}}{10} \end{bmatrix}$

g $\left(-\frac{\sqrt{17}}{17}, \frac{4\sqrt{17}}{17} \right)$

$\mathbf{p} = (-1, 4)$, $|\mathbf{p}| = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$,

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{\sqrt{17}}(-1, 4) = \left(-\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right) = \left(-\frac{\sqrt{17}}{17}, \frac{4\sqrt{17}}{17} \right)$

h $\begin{bmatrix} -\frac{5}{13} \\ \frac{12}{13} \end{bmatrix}$

$\mathbf{p} = \begin{bmatrix} -5 \\ 12 \end{bmatrix}$, $|\mathbf{p}| = \sqrt{(-5)^2 + 12^2} = 13$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{13} \begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -\frac{5}{13} \\ \frac{12}{13} \end{bmatrix}$

Question 4

a $\frac{1}{2}(\sqrt{3},1)$

$\underset{\sim}{\mathbf{p}} = (4, 30°) = (4\cos(30°), 4\sin(30°)) = \left(4 \times \frac{\sqrt{3}}{2}, 4 \times \frac{1}{2}\right) = (2\sqrt{3}, 2)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 4$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{4}(2\sqrt{3}, 2) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b $\frac{1}{2}(1,\sqrt{3})$

$\underset{\sim}{\mathbf{p}} = (7, 60°) = (7\cos(60°), 7\sin(60°)) = \left(\frac{7}{2}, \frac{7\sqrt{3}}{2}\right)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 7$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{7}\left(\frac{7}{2}, \frac{7\sqrt{3}}{2}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

c $(0,1)$

$\underset{\sim}{\mathbf{p}} = (12, 90°) = (12\cos(90°), 12\sin(90°)) = (0, 12)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 12$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{12}(0, 12) = (0, 1)$

d $(-1,0)$

$\underset{\sim}{\mathbf{p}} = (6, 180°) = (6\cos(180°), 6\sin(180°)) = (-6, 0)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 6$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{6}(-6, 0) = (-1, 0)$

e $(0, -1)$

$\underset{\sim}{\mathbf{p}} = (11, 270°) = (11\cos(270°), 11\sin(270°)) = (0, -11)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 11$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{11}(0, -11) = (0, -1)$

f $(1, 0)$

$\underset{\sim}{\mathbf{p}} = (8, 0°) = (8\cos(0°), 8\sin(0°)) = (8, 0)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 8$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{8}(8, 0) = (1, 0)$

g $(0.669, 0.743)$

$\underset{\sim}{\mathbf{p}} = (4, 48°) = (4\cos(48°), 4\sin(48°)) = (2.677, 2.973)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 4$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{4}(2.677, 2.973) = (0.669, 0.743)$

h $(-0.914, 0.407)$

$\underset{\sim}{\mathbf{p}} = (9, 156°) = (9\cos(156°), 9\sin(156°)) = (-8.222, 3.661)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 9$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{9}(-8.222, 3.661) = (-0.914, 0.407)$

i $(0.423, -0.906)$

$\underset{\sim}{\mathbf{p}} = (6, 295°) = (6\cos(295°), 6\sin(295°)) = (2.536, -5.438)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 6$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{6}(2.536, -5.438) = (0.423, -0.906)$

j $(0.848, -0.530)$

$\underset{\sim}{\mathbf{p}} = (13, 328°) = (13\cos(328°), 13\sin(328°)) = (11.025, -6.889)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 13$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{13}(11.025, -6.889) = (0.848, -0.530)$

k $(0.616, -0.788)$

$\underset{\sim}{\mathbf{p}} = (10, -52°) = (10\cos(-52°), 10\sin(-52°)) = (6.157, -7.880)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 10$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{10}(6.156, -7.880) = (0.616, -0.788)$

l $(-0.799, -0.602)$

$\underset{\sim}{\mathbf{p}} = (15, -143°) = (15\cos(-143°), 15\sin(-143°)) = (-11.980, -9.027)$

$\left|\underset{\sim}{\mathbf{p}}\right| = 15$

$\hat{\underset{\sim}{\mathbf{p}}} = \frac{1}{15}(-11.980, -9.027) = (-0.799, -0.602)$

Question 5

a $5\underset{\sim}{i} - 3\underset{\sim}{j}$

$(5, -3) = (5, 0) + (0, -3) = 5(1, 0) + [-3(0, 1)] = 5\underset{\sim}{i} + (-3\underset{\sim}{j}) = 5\underset{\sim}{i} - 3\underset{\sim}{j}$

b $-6\underset{\sim}{i} + 4\underset{\sim}{j}$

$(-6, 4) = (-6, 0) + (0, 4) = -6(1, 0) + 4(0, 1) = -6\underset{\sim}{i} + 4\underset{\sim}{j}$

c $-4\underset{\sim}{i} - 7\underset{\sim}{j}$

$(-4, -7) = (-4, 0) + (0, -7) = -4(1, 0) + [-7(0, 1)] = -4\underset{\sim}{i} + (-7\underset{\sim}{j}) = -4\underset{\sim}{i} - 7\underset{\sim}{j}$

d $6\underset{\sim}{i}+5\underset{\sim}{j}$

$(6,5)=(6,0)+(0,5)=6(1,0)+5(0,1)=6\underset{\sim}{i}+5\underset{\sim}{j}$

e $-3.2\underset{\sim}{i}-9.4\underset{\sim}{j}$

$(-3.2,-9.4)=(-3.2,0)+(0,-9.4)=-3.2(1,0)+[-9.4(0,1)]=-3.2\underset{\sim}{i}+\left(-9.4\underset{\sim}{j}\right)=-3.2\underset{\sim}{i}-9.4\underset{\sim}{j}$

f $\underset{\sim}{i}-4\underset{\sim}{j}$

$\begin{bmatrix}1\\-4\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}0\\-4\end{bmatrix}=1\begin{bmatrix}1\\0\end{bmatrix}+\left(-4\begin{bmatrix}0\\1\end{bmatrix}\right)=1\underset{\sim}{i}+\left(-4\underset{\sim}{j}\right)=\underset{\sim}{i}-4\underset{\sim}{j}$

g $-2\underset{\sim}{i}-8\underset{\sim}{j}$

$\begin{bmatrix}-2\\-8\end{bmatrix}=\begin{bmatrix}-2\\0\end{bmatrix}+\begin{bmatrix}0\\-8\end{bmatrix}=-2\begin{bmatrix}1\\0\end{bmatrix}+\left(-8\begin{bmatrix}0\\1\end{bmatrix}\right)=-2\underset{\sim}{i}+\left(-8\underset{\sim}{j}\right)=-2\underset{\sim}{i}-8\underset{\sim}{j}$

h $-3\underset{\sim}{i}+5\underset{\sim}{j}$

$\begin{bmatrix}-3\\5\end{bmatrix}=\begin{bmatrix}-3\\0\end{bmatrix}+\begin{bmatrix}0\\5\end{bmatrix}=-3\begin{bmatrix}1\\0\end{bmatrix}+5\begin{bmatrix}0\\1\end{bmatrix}=-3\underset{\sim}{i}+5\underset{\sim}{j}$

i $7.59\underset{\sim}{i}+3.68\underset{\sim}{j}$

$\begin{bmatrix}7.59\\3.68\end{bmatrix}=\begin{bmatrix}7.59\\0\end{bmatrix}+\begin{bmatrix}0\\3.68\end{bmatrix}=7.59\begin{bmatrix}1\\0\end{bmatrix}+3.68\begin{bmatrix}0\\1\end{bmatrix}=7.59\underset{\sim}{i}+3.68\underset{\sim}{j}$

j $0.07\underset{\sim}{i}+0.19\underset{\sim}{j}$

$\begin{bmatrix}0.07\\0.19\end{bmatrix}=\begin{bmatrix}0.07\\0\end{bmatrix}+\begin{bmatrix}0\\0.19\end{bmatrix}=0.07\begin{bmatrix}1\\0\end{bmatrix}+1.19\begin{bmatrix}0\\1\end{bmatrix}=0.17\underset{\sim}{i}+1.19\underset{\sim}{j}$

Question 6

a $1.93\underset{\sim}{i} - 6.73\underset{\sim}{j}$

$(7, -74°) = (7\cos(-74°), 7\sin(-74°)) = (1.93, -6.73)$

$$\begin{aligned}(1.93, -6.73) &= (1.93, 0) + (0, -6.73) \\ &= 1.93(1,0) + \left[-6.73(0,1)\right] \\ &= 1.93\underset{\sim}{i} + \left(-6.73\underset{\sim}{j}\right) \\ &= 1.93\underset{\sim}{i} - 6.73\underset{\sim}{j}\end{aligned}$$

b $-5.16\underset{\sim}{i} + 7.37\underset{\sim}{j}$

$(9, 125°) = (9\cos(125°), 9\sin(125°)) = (-5.16, 7.37)$

$$\begin{aligned}(-5.16, 7.37) &= (-5.16, 0) + (0, 7.37) \\ &= -5.16(1,0) + 7.37(0,1) \\ &= -5.16\underset{\sim}{i} + 7.37\underset{\sim}{j}\end{aligned}$$

c $9.53\underset{\sim}{i} - 5.50\underset{\sim}{j}$

$(11, 330°) = (11\cos(330°), 11\sin(330°)) = (9.53, -5.50)$

$$\begin{aligned}(-5.16, -5.50) &= (9.53, 0) + (0, -5.50) \\ &= 9.53(1,0) + \left[-5.50(0,1)\right] \\ &= 9.53\underset{\sim}{i} + \left(-5.50\underset{\sim}{j}\right) \\ &= 9.53\underset{\sim}{i} - 5.50\underset{\sim}{j}\end{aligned}$$

d $-8\underset{\sim}{i} - 13.86\underset{\sim}{j}$

$(16, 240°) = (16\cos(240°), 16\sin(240°)) = (-8, -13.86)$

$$\begin{aligned}(-8, -13.86) &= (-8, 0) + (0, -13.86) \\ &= -8(1,0) + \left[-13.86(0,1)\right] \\ &= -8\underset{\sim}{i} + \left(-13.86\underset{\sim}{j}\right) \\ &= -8\underset{\sim}{i} - 13.86\underset{\sim}{j}\end{aligned}$$

e $-2.33\underset{\sim}{i} - 4.20\underset{\sim}{j}$

$(4.8, -119°) = (4.8\cos(-119°), 4.8\sin(-119°)) = (-2.33, -4.20)$

$$\begin{aligned}(-2.33, -4.20) &= (-2.33, 0) + (0, -4.20) \\ &= -2.33(1,0) + \left[-4.20(0,1)\right] \\ &= -2.33\underset{\sim}{i} + \left(-4.20\underset{\sim}{j}\right) \\ &= -2.33\underset{\sim}{i} - 4.20\underset{\sim}{j}\end{aligned}$$

f $0.70\underset{\sim}{i} + 7.97\underset{\sim}{j}$

$(8, 85°) = (8\cos(85°), 8\sin(85°)) = (0.70, 7.97)$

$$\begin{aligned}(0.70, 7.97) &= (0.70, 0) + (0, 7.97) \\ &= 0.70(1,0) + 7.97(0,1) \\ &= 0.70\underset{\sim}{i} + 7.97\underset{\sim}{j}\end{aligned}$$

g $-2.78\underset{\sim}{i} + 1.12\underset{\sim}{j}$

$(3, 158°) = (3\cos(158°), 3\sin(158°)) = (-2.78, 1.12)$

$$\begin{aligned}(-2.78, 1.12) &= (-2.78, 0) + (0, 1.12) \\ &= -2.78(1,0) + 1.12(0,1) \\ &= -2.78\underset{\sim}{i} + 1.12\underset{\sim}{j}\end{aligned}$$

h $0.52\underset{\sim}{i} - 9.99\underset{\sim}{j}$

$(10, -87°) = (10\cos(-87°), 10\sin(-87°)) = (0.52, -9.99)$

$$\begin{aligned}(0.52, -9.99) &= (0.52, 0) + (0, -9.99) \\ &= 0.52(1,0) + \left[-9.99(0,1)\right] \\ &= 0.52\underset{\sim}{i} - 9.99\underset{\sim}{j}\end{aligned}$$

i $-3.88\underset{\sim}{i} + 14.49\underset{\sim}{j}$

$(15, -255°) = (15\cos(-255°), 15\sin(-255°)) = (-3.88, 14.49)$

$$\begin{aligned}(0.52, -9.99) &= (-3.88, 0) + (0, 14.49) \\ &= -3.88(1,0) + 14.49(0,1) \\ &= -3.88\underset{\sim}{i} + 14.49\underset{\sim}{j}\end{aligned}$$

j $-9.70\underset{\sim}{i} - 17.49\underset{\sim}{j}$

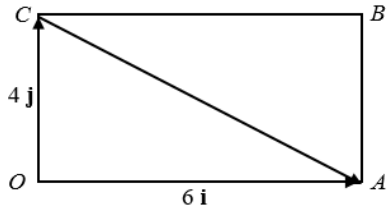
$(20, -119°) = (20\cos(-119°), 20\sin(-119°)) = (-9.70, -17.49)$

$$\begin{aligned}(-9.70, -17.49) &= (-9.70, 0) + (0, -17.49) \\ &= -9.70(1, 0) + [-17.49(0, 1)] \\ &= -9.70\underset{\sim}{i} - 17.49\underset{\sim}{j}\end{aligned}$$

Question 7

a $\mathbf{OC} + \mathbf{CA} = \mathbf{OA}$

$\mathbf{CA} = \mathbf{OA} - \mathbf{OC} = 6i - 4j$



b $\mathbf{OC} + \mathbf{CA} = \mathbf{OA}$

$\mathbf{CA} = \mathbf{OA} - \mathbf{OC}$

$\mathbf{AC} = -\mathbf{CA} = -(\mathbf{OA} - \mathbf{OC}) = -\mathbf{OA} + \mathbf{OC} = -6i + 4j$

Question 8

$\left(\frac{5}{13}, -\frac{12}{13}\right)$

Let $\underset{\sim}{p} = -5\underset{\sim}{i} + 12\underset{\sim}{j}$ and let $\underset{\sim}{q} = -\underset{\sim}{p} = 5\underset{\sim}{i} - 12\underset{\sim}{j}$

$|\underset{\sim}{q}| = \sqrt{5^2 + (-12)^2} = 13$

$\hat{\underset{\sim}{q}} = \frac{\underset{\sim}{q}}{|\underset{\sim}{q}|} = \frac{1}{13}(5\underset{\sim}{i} - 12\underset{\sim}{j})$

Question 9

$(-9, 12)$

$\underset{\sim}{h} = -6\underset{\sim}{i} + 8\underset{\sim}{j}$

$|\underset{\sim}{h}| = \sqrt{(-6)^2 + 8^2} = 10$

$\hat{\underset{\sim}{h}} = \frac{\underset{\sim}{h}}{|\underset{\sim}{h}|} = \frac{1}{10}(-6\underset{\sim}{i} + 8\underset{\sim}{j})$

$15\hat{\underset{\sim}{h}}$ is a vector of magnitude 15 in the same direction as $\underset{\sim}{h}$.

$15\hat{\underset{\sim}{h}} = 15 \times \frac{1}{10}(-6\underset{\sim}{i} + 8\underset{\sim}{j}) = -9\underset{\sim}{i} + 12\underset{\sim}{j}$

Question 1

The tail of each vector is translated to the origin.

For example, the tail of vector **a** is (2, 1) and its head is at (7, 4).

Translate the tail 2 units left and 1 unit down. (2 – 2, 1 – 1) to (0, 0).

The position vector becomes (7 – 2, 4 – 1) or (5, 3).

a = (5, 3), **b** = (5, 7), **c** = (8, –5), **d** = (–5, 2), **e** = (–3, –8), **f** = (–5, 5) and **g** = (–5, –4).

a (5, 3) + (5, 7) = (5 + 5, 3 + 7) = (10, 10)

b (8, –5) + (–3, –8) = (8 + (–3), –5 – 8) = (5, –13)

c (–5, 2) + (5, 7) = (–5 + 5, 2 + 7) = (0, 9)

d (–5, 5) + (5, 3) = (–5 + 5, 5 + 3) = (0, 8)

e (–5, –4) + (–5, 5) = (–5 + (–5), –4 + 5) = (–10, 1)

f (5, 7) + (8, –5) = (5 + 8, 7 + (–5)) = (13, 2)

g (–5, 2) + (–5, –4) = (–5 + (–5), 2 + (–4)) = (–10, –2)

Question 2

a (–2, 13)

(3, 6) + (–5, 7) = (3 + (–5), 6 + 7) = (–2, 13)

b (–20, 1)

(–12, 7) + (–8, –6) = (–12 + (–8), 7 + (–6)) = (–20, 1)

c (6.1, 3.7)

(–11.5, –9.8) + (17.6, 13.5) = (–11.5 + 17.6, –9.8 + 13.5) = (6.1, 3.7)

d (13.16, –10.19)

(7.05, –4.32) + (6.11, –5.87) = (7.05 + 6.11, –4.32 + (–5.87)) = (13.16, –10.19)

e $\left(-1\frac{1}{4}, \frac{5}{6}\right)$

$$\left(-4\frac{1}{2}, 6\frac{2}{3}\right) + \left(3\frac{1}{4}, -5\frac{5}{6}\right) = \left(-4\frac{1}{2} + 3\frac{1}{4}, 6\frac{2}{3} + -5\frac{5}{6}\right) = \left(-1\frac{1}{4}, \frac{5}{6}\right)$$

f $\left(-5\frac{1}{10}, 2\frac{7}{12}\right)$

$$\left(2\frac{3}{5}, -1\frac{3}{4}\right) + \left(-7\frac{7}{10}, 4\frac{1}{3}\right) = \left(2\frac{3}{5} + -7\frac{7}{10}, -1\frac{3}{4} + 4\frac{1}{3}\right) = \left(-5\frac{1}{10}, 2\frac{7}{12}\right)$$

Question 3

a $\begin{bmatrix} 9 \\ -7 \end{bmatrix} + \begin{bmatrix} -3 \\ 12 \end{bmatrix} = \begin{bmatrix} 9 + -3 \\ -7 + 12 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

b $\begin{bmatrix} -4 \\ -8 \end{bmatrix} + \begin{bmatrix} 11 \\ 7 \end{bmatrix} = \begin{bmatrix} -4 + 11 \\ -8 + 7 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$

c $\begin{bmatrix} -8 \\ 10 \end{bmatrix} + \begin{bmatrix} 13 \\ -6 \end{bmatrix} = \begin{bmatrix} -8 + 13 \\ 10 + (-6) \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

d $\begin{bmatrix} 3.9 \\ 5.7 \end{bmatrix} + \begin{bmatrix} 7.4 \\ -2.6 \end{bmatrix} = \begin{bmatrix} 3.9 + 7.4 \\ 5.7 + (-2.6) \end{bmatrix} = \begin{bmatrix} 11.3 \\ 3.1 \end{bmatrix}$

e $\begin{bmatrix} 7.09 \\ 5.46 \end{bmatrix} + \begin{bmatrix} -4.71 \\ -3.28 \end{bmatrix} = \begin{bmatrix} 7.09 + (-4.71) \\ 5.46 + (-3.28) \end{bmatrix} = \begin{bmatrix} 2.38 \\ 2.18 \end{bmatrix}$

f $\begin{bmatrix} 2\frac{1}{3} \\ -4\frac{1}{2} \end{bmatrix} + \begin{bmatrix} -3\frac{1}{6} \\ -2\frac{1}{4} \end{bmatrix} = \begin{bmatrix} 2\frac{1}{3} + \left(-3\frac{1}{6}\right) \\ -4\frac{1}{2} + \left(-2\frac{1}{4}\right) \end{bmatrix} = \begin{bmatrix} -\frac{5}{6} \\ -6\frac{3}{4} \end{bmatrix}$

Question 4

a $6\underset{\sim}{i} + \underset{\sim}{j}$

$$(2\underset{\sim}{i} - 10\underset{\sim}{j}) + (4\underset{\sim}{i} + 11\underset{\sim}{j}) = (2\underset{\sim}{i} + 4\underset{\sim}{i}) + (-10\underset{\sim}{j} + 11\underset{\sim}{j})$$
$$= 6\underset{\sim}{i} + 1\underset{\sim}{j} = 6\underset{\sim}{i} + \underset{\sim}{j}$$

b $13\underset{\sim}{i} + 5\underset{\sim}{j}$

$$(7\underset{\sim}{i} + 9\underset{\sim}{j}) + (6\underset{\sim}{i} - 4\underset{\sim}{j}) = (7\underset{\sim}{i} + 6\underset{\sim}{i}) + (9\underset{\sim}{j} - 4\underset{\sim}{j})$$
$$= 13\underset{\sim}{i} + 5\underset{\sim}{j}$$

c $-2.2\underset{\sim}{i} - 15.0\underset{\sim}{j}$

$$(-8.4\underset{\sim}{i} - 5.7\underset{\sim}{j}) + (6.2\underset{\sim}{i} - 9.3\underset{\sim}{j}) = (-8.4\underset{\sim}{i} + 6.2\underset{\sim}{i}) + (-5.7\underset{\sim}{j} - 9.3\underset{\sim}{j})$$
$$= -2.2\underset{\sim}{i} - 15.0\underset{\sim}{j}$$

d $-2.11\underset{\sim}{i} - 2.92\underset{\sim}{j}$

$$(-5.67\underset{\sim}{i} + 4.92\underset{\sim}{j}) + (3.56\underset{\sim}{i} - 7.84\underset{\sim}{j}) = (-5.67\underset{\sim}{i} + 3.56\underset{\sim}{i}) + (4.92\underset{\sim}{j} - 7.84\underset{\sim}{j})$$
$$= -2.11\underset{\sim}{i} - 2.92\underset{\sim}{j}$$

e $8.23\underset{\sim}{i} - 19.31\underset{\sim}{j}$

$$(1.06\underset{\sim}{i} - 9.22\underset{\sim}{j}) + (7.17\underset{\sim}{i} - 10.09\underset{\sim}{j}) = (1.06\underset{\sim}{i} + 7.17\underset{\sim}{i}) + (-9.22\underset{\sim}{j} - 10.09\underset{\sim}{j})$$
$$= 8.23\underset{\sim}{i} - 19.31\underset{\sim}{j}$$

f $-2\frac{2}{9}\underset{\sim}{i} + -3\frac{7}{12}\underset{\sim}{j}$

$$\left(3\frac{1}{3}\underset{\sim}{i} + 2\frac{3}{4}\underset{\sim}{j}\right) + \left(-5\frac{5}{9}\underset{\sim}{i} - 6\frac{1}{3}\underset{\sim}{j}\right) = \left(3\frac{1}{3}\underset{\sim}{i} - 5\frac{5}{9}\underset{\sim}{i}\right) + \left(2\frac{3}{4}\underset{\sim}{j} - 6\frac{1}{3}\underset{\sim}{j}\right)$$
$$= -2\frac{2}{9}\underset{\sim}{i} + -3\frac{7}{12}\underset{\sim}{j}$$

Question 5

a $(-4, 5)$

$(2, 4) + (-6, 1) = (2 + (-6), 4 + 1) = (-4, 5)$

b $(0, 10)$

$(-7, 8) + (2, 4) + (5, -2) = (-7 + 2 + 5, 8 + 4 + (-2)) = (0, 10)$

c $(5, 20)$

$5(1, 4) = (5 \times 1, 5 \times 4) = (5, 20)$

d $(12, -2)$

$-2(-6, 1) = (-2 \times (-6), -2 \times 1) = (12, -2)$

e $(-9, 14)$

$3(1, 4) + 2(-6, 1) = (3, 12) + (-12, 2) = (3 + (-12), 12 + 2) = (-9, 14)$

f $(-30, 24)$

$4(-7, 8) - 2(1, 4) = (-28, 32) - (2, 8) = (-28 - 2, 32 - 8) = (-30, 24)$

g $(5, -1)$

$(1, 4) - (2, 4) - (-6, 1) = (1 - 2 - (-6), 4 - 4 - 1) = (5, -1)$

h $(0, 0)$

$7(2, 4) - 7(1, 4) + (5, -2) + 2(-6, 1)$

$= (14, 28) - (7, 28) + (5, -2) + (-12, 2)$

$= (14 - 7 + 5 - 12, 28 - 28 - 2 + 2) = (0, 0)$

i $(-34, 69)$

$7(1, 4) + 5(-7, 8) + (-6, 1)$

$= (7, 28) + (-35, 40) + (-6, 1)$

$= (7 - 35 - 6, 28 + 40 + 1) = (-34, 69)$

j $(-4, -32)$

$2(5, -2) - 7(2, 4)$

$= (10, -4) - (14, 28)$

$= (10 - 14, -4 - 28) = (-4, -32)$

Question 6

a $4\underset{\sim}{i} - 5\underset{\sim}{j}$ or $(4, -5)$

$3(2\underset{\sim}{i} + 3\underset{\sim}{j}) - (\underset{\sim}{i} - 4\underset{\sim}{j}) = 6\underset{\sim}{i} + 9\underset{\sim}{j} - \underset{\sim}{i} + 4\underset{\sim}{j} = 5\underset{\sim}{i} + 13\underset{\sim}{j}$

b $5\underset{\sim}{i} + 13\underset{\sim}{j}$ or $(5, 13)$

$2\underset{\sim}{i} + 3\underset{\sim}{j} + 2(\underset{\sim}{i} - 4\underset{\sim}{j}) = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{i} - 8\underset{\sim}{j} = 4\underset{\sim}{i} - 5\underset{\sim}{j}$

c $9\underset{\sim}{i} + 7\underset{\sim}{j}$ or $(9, 7)$

$3(2\underset{\sim}{i} + 3\underset{\sim}{j}) - (-3\underset{\sim}{i} + 2\underset{\sim}{j}) = 6\underset{\sim}{i} + 9\underset{\sim}{j} + 3\underset{\sim}{i} - 2\underset{\sim}{j} = 9\underset{\sim}{i} + 7\underset{\sim}{j}$

d $-9\underset{\sim}{i} + 16\underset{\sim}{j}$ or $(-9, 16)$

$2(-3\underset{\sim}{i} + 2\underset{\sim}{j}) - 3(\underset{\sim}{i} - 4\underset{\sim}{j}) = -6\underset{\sim}{i} + 4\underset{\sim}{j} - 3\underset{\sim}{i} + 12\underset{\sim}{j} = -9\underset{\sim}{i} + 16\underset{\sim}{j}$

e $\underset{\sim}{j}$ or $(0, 1)$

$(2\underset{\sim}{i} + 3\underset{\sim}{j}) + (\underset{\sim}{i} - 4\underset{\sim}{j}) + (-3\underset{\sim}{i} + 2\underset{\sim}{j}) = (2\underset{\sim}{i} + \underset{\sim}{i} - 3\underset{\sim}{i}) + (3\underset{\sim}{j} - 4\underset{\sim}{j} + 2\underset{\sim}{j}) = 0\underset{\sim}{i} + 1\underset{\sim}{j} = \underset{\sim}{j}$

f $5\underset{\sim}{i} - 10\underset{\sim}{j}$ or $(5, -10)$

$2(\underset{\sim}{i} - 4\underset{\sim}{j}) - (-3\underset{\sim}{i} + 2\underset{\sim}{j}) = 2\underset{\sim}{i} - 8\underset{\sim}{j} + 3\underset{\sim}{i} - 2\underset{\sim}{j} = 5\underset{\sim}{i} - 10\underset{\sim}{j}$

g $5\underset{\sim}{i} + 3\underset{\sim}{j}$ or $(5, 3)$

$3(2\underset{\sim}{i} + 3\underset{\sim}{j}) + 2(\underset{\sim}{i} - 4\underset{\sim}{j}) + (-3\underset{\sim}{i} + 2\underset{\sim}{j}) = 6\underset{\sim}{i} + 9\underset{\sim}{j} + 2\underset{\sim}{i} - 8\underset{\sim}{j} - 3\underset{\sim}{i} + 2\underset{\sim}{j} = 5\underset{\sim}{i} + 3\underset{\sim}{j}$

h $-8\underset{\sim}{i} + 14\underset{\sim}{j}$ or $(-8, 14)$

$4(-3\underset{\sim}{i} + 2\underset{\sim}{j}) + 2(2\underset{\sim}{i} + 3\underset{\sim}{j}) = -12\underset{\sim}{i} + 8\underset{\sim}{j} + 4\underset{\sim}{i} + 6\underset{\sim}{j} = -8\underset{\sim}{i} + 14\underset{\sim}{j}$

i $-14\underset{\sim}{i} + 5\underset{\sim}{j}$ or $(-14, 5)$

$4(-3\underset{\sim}{i} + 2\underset{\sim}{j}) - (2\underset{\sim}{i} + 3\underset{\sim}{j}) = -12\underset{\sim}{i} + 8\underset{\sim}{j} - 2\underset{\sim}{i} - 3\underset{\sim}{j} = -14\underset{\sim}{i} + 5\underset{\sim}{j}$

j $-12\underset{\sim}{i} + 7\underset{\sim}{j}$ or $(-12, 7)$

$3(-3\underset{\sim}{i} + 2\underset{\sim}{j}) - (2\underset{\sim}{i} + 3\underset{\sim}{j}) - (\underset{\sim}{i} - 4\underset{\sim}{j})$
$= -9\underset{\sim}{i} + 6\underset{\sim}{j} - 2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{i} + 4\underset{\sim}{j}$
$= -12\underset{\sim}{i} + 7\underset{\sim}{j}$

Question 7

A graphics calculator can be used, or by calculation as shown here.

a (14.33, 41.1°)

$$[6\cos(20°)\underset{\sim}{i} + 6\sin(20°)\underset{\sim}{j}] + [9\cos(55°)\underset{\sim}{i} + 9\sin(55°)\underset{\sim}{j}]$$
$$= (5.64\underset{\sim}{i} + 2.05\underset{\sim}{j}) + (5.16\underset{\sim}{i} + 7.37\underset{\sim}{j})$$
$$= 10.8\underset{\sim}{i} + 9.42\underset{\sim}{j}$$

magnitude: $\sqrt{10.8^2 + 9.42^2} = 14.33$

angle: $\tan^{-1}\left(\frac{9.42}{10.8}\right) = 41.1°$

b (38.65, 104.6°)

$$[25\cos(120°)\underset{\sim}{i} + 25\sin(120°)\underset{\sim}{j}] + [16\cos(80°)\underset{\sim}{i} + 16\sin(80°)\underset{\sim}{j}]$$
$$= (-12.5\underset{\sim}{i} + 21.65\underset{\sim}{j}) + (2.78\underset{\sim}{i} + 15.76\underset{\sim}{j})$$
$$= -9.72\underset{\sim}{i} + 37.41\underset{\sim}{j}$$

magnitude: $\sqrt{(-9.72)^2 + 37.41^2} = 38.65$

angle: $\tan^{-1}\left(\frac{37.41}{-9.72}\right) = -75.44°$

The vector is in the 2nd quadrant, so the angle is 180° – 75.44° = 104.6°

c (9.32, 324.9°)

$$[7\cos(30°)\underset{\sim}{i} + 7\sin(30°)\underset{\sim}{j}] - [9\cos(100°)\underset{\sim}{i} + 9\sin(100°)\underset{\sim}{j}]$$
$$= (6.06\underset{\sim}{i} + 3.5\underset{\sim}{j}) - (-1.56\underset{\sim}{i} + 8.86\underset{\sim}{j})$$
$$= 7.62\underset{\sim}{i} - 5.36\underset{\sim}{j}$$

magnitude: $\sqrt{7.62^2 + (-5.36)^2} = 9.32$

angle: $\tan^{-1}\left(\frac{-5.36}{7.62}\right) = -35.12°$

The vector is in the 4th quadrant, so the angle is 360° – 35.12° = 324.9°

d (173.28, 271.7°)

$$[105\cos(300°)\underset{\sim}{i} + 105\sin(300°)\underset{\sim}{j}] - [95\cos(60°)\underset{\sim}{i} + 95\sin(60°)\underset{\sim}{j}]$$
$$= (52.5\underset{\sim}{i} - 90.93\underset{\sim}{j}) - (47.5\underset{\sim}{i} + 82.27\underset{\sim}{j})$$
$$= 5\underset{\sim}{i} - 173.2\underset{\sim}{j}$$

magnitude: $\sqrt{5^2 + (-173.2)^2} = 173.28$

angle: $\tan^{-1}\left(\frac{-173.2}{5}\right) = -88.35°$

The vector is in the 4th quadrant, so the angle is 360° – 88.35° = 271.7°

e (6, 130°)

$$[6\cos(70°)\underset{\sim}{i} + 6\sin(70°)\underset{\sim}{j}] - [6\cos(10°)\underset{\sim}{i} + 6\sin(10°)\underset{\sim}{j}]$$
$$= (2.05\underset{\sim}{i} + 5.64\underset{\sim}{j}) - (5.91\underset{\sim}{i} + 1.04\underset{\sim}{j})$$
$$= -3.86\underset{\sim}{i} + 4.6\underset{\sim}{j}$$

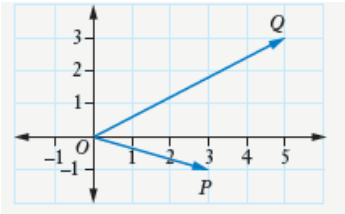
magnitude: $\sqrt{(-3.86)^2 + 4.6^2} = 6.00$

angle: $\tan^{-1}\left(\frac{4.6}{-3.86}\right) = -50.0°$

The vector is in the 2nd quadrant, so the angle is 180° – 50° = 130°

Question 8

a



b $\mathbf{PQ} = \mathbf{PO} + \mathbf{OQ}$

$= -\mathbf{OP} + \mathbf{OQ}$

$= -(3i - j) + (5i + 3j)$

$= -3i + j + 5i + 3j$

$= 2i + 4j$

c $\mathbf{OM} = \mathbf{OP} + \frac{1}{2}\mathbf{PQ} = 3i - j + \frac{1}{2}(2i + 4j) = 3i + i - j + 2j = 4i + j$

Question 9

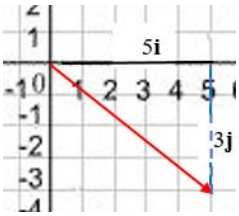
$$k(\mathbf{p} + \mathbf{q}) = k[(x_1, y_1) + (x_2, y_2)]$$
$$= k(x_1 + x_2, y_1 + y_2)$$
$$= (kx_1 + kx_2, ky_1 + ky_2)$$

$$k\mathbf{p} + k\mathbf{q} = k(x_1, y_1) + k(x_2, y_2)$$
$$= (kx_1, ky_1) + (kx_2, ky_2)$$
$$= (kx_1 + kx_2, ky_1 + ky_2)$$

Exercise 1.08 Resolution of vectors

Question 1

a $5\mathbf{i} - 3\mathbf{j}$



b $-6\mathbf{i} + 4\mathbf{j}$

c $-4\mathbf{i} - 7\mathbf{j}$

d $6\mathbf{i} + 5\mathbf{j}$

e $-3.2\mathbf{i} - 9.4\mathbf{j}$

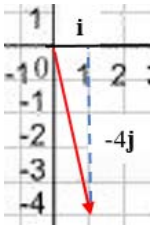
f $2\sqrt{3}\,\mathbf{i} + \sqrt{5}\,\mathbf{j}$

g $\frac{2}{3}\mathbf{i} + \left(-\frac{4}{9}\right)\mathbf{j}$ or $\frac{2}{3}\mathbf{i} - \frac{4}{9}\mathbf{j}$

h $-5\sqrt{7}\,\mathbf{i} + 6\sqrt{2}\,\mathbf{j}$

Question 2

a $\mathbf{i} - 4\mathbf{j}$



b $-2\mathbf{i} - 8\mathbf{j}$

c $-3\mathbf{i} + 5\mathbf{j}$

d $7.59\mathbf{i} + 3.68\mathbf{j}$

e $0.07\mathbf{i} + 0.19\mathbf{j}$

f $-3\sqrt{5}\,\mathbf{i} + 4\sqrt{7}\,\mathbf{j}$

g $\frac{6}{11}\mathbf{i} + \left(-\frac{5}{4}\right)\mathbf{j}$ or $\frac{6}{11}\mathbf{i} - \frac{5}{4}\mathbf{j}$

h $-9\sqrt{11}\,\mathbf{i} + \left(-7\sqrt{15}\right)\mathbf{j}$ or $-9\sqrt{11}\,\mathbf{i} - 7\sqrt{15}\,\mathbf{j}$

Question 3

a $1.93\mathbf{i} - 6.73\mathbf{j}$

$7 \cos(-74°)\,\mathbf{i} + 7 \sin(-74°)\,\mathbf{j} = 1.93\mathbf{i} - 6.73\mathbf{j}$

b $-5.16\mathbf{i} + 7.37\mathbf{j}$

$9 \cos(125°)\,\mathbf{i} + 9 \sin(125°)\,\mathbf{j} = -5.16\mathbf{i} + 7.37\mathbf{j}$

c $9.53\mathbf{i} - 5.5\mathbf{j}$

$11 \cos(330°)\,\mathbf{i} + 11 \sin(330°)\,\mathbf{j} = 9.53\mathbf{i} - 5.5\mathbf{j}$

d $-8\mathbf{i} - 13.86\mathbf{j}$

$16 \cos(240°)\,\mathbf{i} + 16 \sin(240°)\,\mathbf{j} = -8\mathbf{i} - 13.86\mathbf{j}$

e $-2.33\mathbf{i} - 4.20\mathbf{j}$

$4.8 \cos(-119°)\,\mathbf{i} + 4.8 \sin(-119°)\,\mathbf{j} = -2.33\mathbf{i} - 4.20\mathbf{j}$

f $7.82\mathbf{i} + 10.38\mathbf{j}$

$13 \cos(53°)\,\mathbf{i} + 13 \sin(53°)\,\mathbf{j} = 7.82\mathbf{i} + 10.38\mathbf{j}$

g $7.75\mathbf{i} - 3.95\mathbf{j}$

$8.7 \cos(-27°)\,\mathbf{i} + 8.7 \sin(-27°)\,\mathbf{j} = 7.75\mathbf{i} - 3.95\mathbf{j}$

h $2.92\mathbf{i} - 10.19\mathbf{j}$

$10.6 \cos(286°)\,\mathbf{i} + 10.6 \sin(286°)\,\mathbf{j} = 2.92\mathbf{i} - 10.19\mathbf{j}$

Question 4

a $46.1\mathbf{i} + 36.4\mathbf{j}$ or (46.1, 36.4)

$\mathbf{a} = 40 \cos(30°)\,\mathbf{i} + 40 \sin(30°)\,\mathbf{j} = 34.64\mathbf{i} + 20\mathbf{j}$

$\mathbf{b} = 20 \cos(55°)\,\mathbf{i} + 20 \sin(55°)\,\mathbf{j} = 11.47\mathbf{i} + 16.38\mathbf{j}$

$\mathbf{a} + \mathbf{b} = (34.64\mathbf{i} + 20\mathbf{j}) + (11.47\mathbf{i} + 16.38\mathbf{j}) = 46.1\mathbf{i} + 36.4\mathbf{j}$ or (46.1, 36.4)

b $6.2\mathbf{i} + 57.8\mathbf{j}$ or (6.2, 57.8)

$\mathbf{a} = 36 \cos(38°)\,\mathbf{i} + 36 \sin(38°)\,\mathbf{i} = 28.37\mathbf{i} + 22.16\mathbf{j}$

$\mathbf{b} = 42 \cos(180° - 58°)\,\mathbf{i} + 42 \sin(180° - 58°)\,\mathbf{j} = -22.26\mathbf{i} + 35.62\mathbf{j}$

$\mathbf{a} + \mathbf{b} = (28.37\mathbf{i} + 22.16\mathbf{j}) + (-22.26\mathbf{i} + 35.62\mathbf{j}) = 6.2\mathbf{i} + 57.8\mathbf{j}$ or (6.2, 57.8)

c $34.8\mathbf{i} - 33.3\mathbf{j}$ or (34.8, –33.3)

$\mathbf{a} = 54 \cos(12°)\,\mathbf{i} + 54 \sin(12°)\,\mathbf{j} = 52.82\mathbf{i} + 11.23\mathbf{j}$

$\mathbf{b} = 48 \cos(180° + 68°)\,\mathbf{i} + 48 \sin(180° + 68°)\,\mathbf{j} = -17.98\mathbf{i} - 44.50\mathbf{j}$

$\mathbf{a} + \mathbf{b} = (52.82\mathbf{i} + 11.23\mathbf{j}) + (-17.98\mathbf{i} - 44.50\mathbf{j}) = 34.8\mathbf{i} - 33.3\mathbf{j}$ or (34.8, –33.3)

Question 5

A graphics calculator can be used, or calculations as shown below.

a (15.48, 217.1°)

$\mathbf{a} = 5\cos(90° + 28°)\,\mathbf{i} + 5\sin(90° + 28°)\,\mathbf{j} = -2.35\mathbf{i} + 4.41\mathbf{j}$

$\mathbf{b} = 17\cos(180° + 54°)\,\mathbf{i} + 17\sin(180° + 54°)\,\mathbf{j} = -9.99\mathbf{i} - 13.75\mathbf{j}$

$\mathbf{a} + \mathbf{b} = (-2.35\mathbf{i} + 4.41\mathbf{j}) + (-9.99\mathbf{i} - 13.75\mathbf{j}) = -12.34\mathbf{i} - 9.34\mathbf{j}$

magnitude: $\sqrt{(-12.34)^2 + (-9.34)^2} = 15.48$

angle: $\tan^{-1}\left(\frac{-9.34}{-12.34}\right) = 37.1°$

The vector is in the 3rd quadrant, so the required angle is 180° + 37.1° = 217.1°

b (13.87, 214.2°)

$\mathbf{a} = 26\cos(270° - 77°)\,\mathbf{i} + 26\sin(270° - 77°)\,\mathbf{j} = -25.33\mathbf{i} - 5.85\mathbf{j}$

$\mathbf{b} = 14\cos(270^0 + 82°)\,\mathbf{i} + 14\sin(270° + 82°)\,\mathbf{j} = 13.86\mathbf{i} - 1.95\mathbf{j}$

$\mathbf{a} + \mathbf{b} = (-25.33\mathbf{i} - 5.85\mathbf{j}) + (13.86\mathbf{i} - 1.95) = -11.47\mathbf{i} - 7.80\mathbf{j}$

magnitude: $\sqrt{(-11.47)^2 + (-7.80)^2} = 13.87$

angle: $\tan^{-1}\left(\frac{-7.80}{-11.47}\right) = 34.2°$

The vector is in the 3rd quadrant, so the required angle is 180° + 34.2° = 214.2°.

c (20.01, 239.6°)

$\mathbf{a} = 22\cos(9°)\,\mathbf{i} + 22\sin(9°)\,\mathbf{j} = 21.73\mathbf{i} + 3.441\mathbf{j}$

$\mathbf{b} = 38\cos(180° + 33°)\,\mathbf{i} + 38\sin(180° + 33°)\,\mathbf{i} = -31.87\,\mathbf{i} - 20.70\mathbf{j}$

$\mathbf{a} + \mathbf{b} = (21.73\mathbf{i} + 3.441\mathbf{j}) + (-31.87\mathbf{i} - 20.70\mathbf{j}) = -10.14\mathbf{i} - 17.25\mathbf{j}$

magnitude: $\sqrt{(-10.14)^2 + (-17.25)^2} = 20.01$

angle: $\tan^{-1}\left(\frac{-17.25}{-10.14}\right) = 59.6°$

The vector is in the 3rd quadrant, so the required angle is 180° + 59.6° = 239.6°.

Question 6

A graphics calculator can be used, or calculate as shown below.

a (63.91, 39.3°)

$\mathbf{a} = (15, 20°) = 15 \cos (20°)\ \mathbf{i} + 15 \sin (20°)\ \mathbf{j} = 14.10\mathbf{i} + 5.13\mathbf{j}$

$\mathbf{b} = (50, 45°) = 50 \cos (45°)\ \mathbf{i} + 50 \sin (45°)\ \mathbf{j} = 35.36\mathbf{i} + 35.36\mathbf{j}$

$\mathbf{a} + \mathbf{b} = 49.45\mathbf{i} + 40.49\mathbf{j}$

magnitude: $\sqrt{49.45^2 + 40.49^2} = 63.91$

angle: $\tan^{-1}\left(\frac{40.49}{49.45}\right) = 39.3°$

b (54.60, 112.1°)

$\mathbf{a} = (37, 100°) = 37 \cos (100°)\ \mathbf{i} + 37 \sin (100°)\ \mathbf{j} = -6.42\mathbf{i} + 36.44\mathbf{j}$

$\mathbf{b} = (20, 135°) = 20 \cos (135°)\ \mathbf{i} + 20 \sin (135°)\ \mathbf{j} = -14.14\mathbf{i} + 14.14\mathbf{j}$

$\mathbf{a} + \mathbf{b} = -20.57\mathbf{i} + 50.58\mathbf{j}$

magnitude: $\sqrt{(-20.57)^2 + 50.58^2} = 54.60$

angle: $\tan^{-1}\left(\frac{50.58}{-20.57}\right) = -67.9°$

The vector is in the 2nd quadrant, so the angle is $180° - 67.9° = 112.1°$

c (55.56, 65.8°)

$\mathbf{a} = (60, -15°) = 60 \cos (-15°)\ \mathbf{i} + 60 \sin (-15°)\ \mathbf{j} = 57.96\mathbf{i} - 15.53\mathbf{j}$

$\mathbf{b} = (75, 118°) = 75 \cos (118°)\ \mathbf{i} + 75 \sin (118°)\ \mathbf{j} = -35.32\mathbf{i} + 66.22\mathbf{j}$

$\mathbf{a} + \mathbf{b} = 22.75\mathbf{i} + 50.69\mathbf{j}$

magnitude: $\sqrt{22.75^2 + 50.69^2} = 55.56$

angle: $\tan^{-1}\left(\frac{50.69}{22.75}\right) = 65.8°$

d (88.23, 247.8°)

$\mathbf{a} = (84, 230°) = 84 \cos (230°)\ \mathbf{i} + 84 \sin (230°)\ \mathbf{j} = –53.99\mathbf{i} – 64.35\mathbf{j}$

$\mathbf{b} = (27, 320°) = 27 \cos (320°)\ \mathbf{i} + 27 \sin (320°)\ \mathbf{j} = 20.68\mathbf{i} – 17.36\mathbf{j}$

$\mathbf{a} + \mathbf{b} = –33.31\mathbf{i} – 81.70\mathbf{j}$

magnitude: $\sqrt{(-33.31)^2 + (-81.70)^2} = 88.23$

angle: $\tan^{-1}\left(\frac{-81.70}{-33.31}\right) = 67.8°$

The vector is in the 3rd quadrant, so the angle is 180° + 67.8° = 247.8°.

e (73.13, 180.5°)

$\mathbf{a} = (25, –130°) = 25 \cos (–130°)\ \mathbf{i} + 25 \sin (–130°)\ \mathbf{j} = –16.07\mathbf{i} – 19.15\mathbf{j}$

$\mathbf{b} = (60, 162°) = 60 \cos (162°)\ \mathbf{i} + 60 \sin (162°)\ \mathbf{j} = –57.06\mathbf{i} + 18.54\mathbf{j}$

$\mathbf{a} + \mathbf{b} = –73.13\mathbf{i} – 0.61\mathbf{j}$

magnitude: $\sqrt{(-73.13)^2 + (-0.61)^2} = 73.13$

angle: $\tan^{-1}\left(\frac{-0.61}{-73.31}\right) = 0.5°$

The vector is in the 3rd quadrant, so the angle is 180° + 0.5° = 180.5°.

f (49.71, 194.7°)

$\mathbf{a} = (44, –126°) = 44 \cos (–126°)\ \mathbf{i} + 44 \sin (–126°)\ \mathbf{j} = –25.86\mathbf{i} – 35.60\mathbf{j}$

$\mathbf{b} = (32, 134°) = 32 \cos (134°)\ \mathbf{i} + 32 \sin (134°)\ \mathbf{j} = –22.23\mathbf{i} + 23.02\mathbf{j}$

$\mathbf{a} + \mathbf{b} = –48.09\mathbf{i} – 12.58\mathbf{j}$

magnitude: $\sqrt{(-48.09)^2 + (-12.58)^2} = 49.71$

angle: $\tan^{-1}\left(\frac{-12.58}{-48.09}\right) = 14.7°$

The vector is in the 3rd quadrant, so the angle is 180° + 14.7° = 194.7°.

Question 7

a (92.95, 39.3°)

Let $\mathbf{a} = (58, 180° - 72°) = (58, 108°) = 58 \cos(108°)\,\mathbf{i} + 58 \sin(108°)\,\mathbf{j}$

$= -17.92\mathbf{i} + 55.16\mathbf{j}$

Let $\mathbf{b} = (45, 26°) = 45 \cos(26°)\,\mathbf{i} + 45 \sin(26°)\,\mathbf{j} = 40.45\mathbf{i} + 19.73\mathbf{j}$

Let $\mathbf{c} = (52, 360° - 18°) = (52, 342°) = 52 \cos(342°)\,\mathbf{i} + 52 \sin(342°)\,\mathbf{j}$

$= 49.45\mathbf{i} - 16.07\mathbf{j}$

$\mathbf{a} + \mathbf{b} + \mathbf{c} = 71.98\mathbf{i} + 58.82\mathbf{j}$

magnitude: $\sqrt{71.98^2 + 58.82^2} = 92.95$

angle: $\tan^{-1}\left(\frac{58.82}{71.98}\right) = 39.3°$

b (33.00, 232.9°)

Let $\mathbf{a} = (15, 180° - 24°) = (15, 156°) = 15 \cos(156°)\,\mathbf{i} + 15 \sin(156°)\,\mathbf{j}$

$= -13.70\mathbf{i} + 6.10\mathbf{j}$

Let $\mathbf{b} = (17, 270° - 48°) = 17 \cos(222°)\,\mathbf{i} + 17 \sin(222°)\,\mathbf{j} = -12.63\mathbf{i} - 11.38\mathbf{j}$

Let $\mathbf{c} = (22, 360° - 73°) = (22, 287°) = 22 \cos(287°)\,\mathbf{i} + 22 \sin(287°)\,\mathbf{j}$

$= 6.43\mathbf{i} - 21.04\mathbf{j}$

$\mathbf{a} + \mathbf{b} + \mathbf{c} = -19.90\mathbf{i} - 26.32\mathbf{j}$

magnitude: $\sqrt{(-19.90)^2 + (-26.32)^2} = 33.00$

angle: $\tan^{-1}\left(\frac{-26.32}{-19.90}\right) = 52.9°$

The vector is in the 3rd quadrant, so the required angle is 180° + 52.9° = 232.9°.

c (110.53, 338.2°)

Let $\mathbf{a}$ = (42, 180° + 64°) = (42, 244°) = 42 cos (244°) $\mathbf{i}$ + 42 sin (244°) $\mathbf{j}$

= –18.41$\mathbf{i}$ – 37.75$\mathbf{j}$

Let $\mathbf{b}$ = (57, 270° + 81°) = (57, 351°) = 57 cos (351°) $\mathbf{i}$ + 57 sin (351°) $\mathbf{j}$

= 56.30$\mathbf{i}$ – 8.92$\mathbf{j}$

Let $\mathbf{c}$ = (65, 90° – 85°) = (65, 5°) = 65 cos (5°) $\mathbf{i}$ + 65 sin (5°) $\mathbf{j}$ = 64.75$\mathbf{i}$ + 5.67$\mathbf{j}$

$\mathbf{a} + \mathbf{b} + \mathbf{c}$ = 102.64$\mathbf{i}$ – 41.00$\mathbf{j}$

magnitude: $\sqrt{102.64^2 + 41.00^2} = 110.53$

angle: $\tan^{-1}\left(\frac{-41.00}{102.64}\right) = -21.77°$

The vector is in the 4th quadrant, so the required angle is 360° – 21.77° = 338.2°

Question 8

(2000, 0°) The resultant force is parallel to the direction of the boat.

Let $\mathbf{a}$ = (2000, 60°) = 2000 cos (60°) $\mathbf{i}$ + 2000 sin (60°) $\mathbf{j}$ = 1000$\mathbf{i}$ + 1732.05$\mathbf{j}$

Let $\mathbf{b}$ = (2000, 360° – 60°) = (2000, 300°) = 2000 cos (300°) $\mathbf{i}$ + 2000 sin (300°) $\mathbf{j}$

= 1000$\mathbf{i}$ – 1732.05$\mathbf{j}$

$\mathbf{a} + \mathbf{b}$ = 2000$\mathbf{i}$ + 0$\mathbf{j}$

magnitude: $\sqrt{2000^2 + 0^2} = 2000$

angle: $\tan^{-1}\left(\frac{0}{2000}\right) = 0°$

Question 9

(47.95, S 12° W)

Let $\mathbf{a} = (36, 90° + 63°) = 36 \cos(153°)\,\mathbf{i} + 36 \sin(153°)\,\mathbf{j}$

$= -32.08\mathbf{i} + 16.34\mathbf{j}$

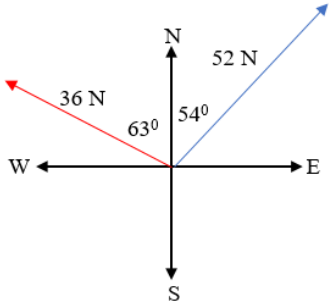
Let $\mathbf{b} = (52, 90° - 54°) = 52 \cos(36°)\,\mathbf{i} + 52 \sin(36°)\,\mathbf{j}$

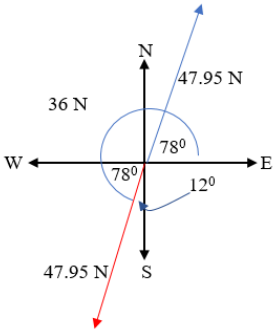
$= 42.07\mathbf{i} + 30.56\mathbf{j}$

$\mathbf{a} + \mathbf{b} = 9.99\mathbf{i} + 46.9\mathbf{j}$

magnitude: $\sqrt{9.99^2 + 46.9^2} = 47.95$

angle: $\tan^{-1}\left(\frac{46.9}{9.99}\right) = 78.0°$





A vector in the opposite direction has direction S 12° W .

Question 10

N 82.8° E

Let $\mathbf{a} = (270, 84°) = 270 \cos(84°)\,\mathbf{i} + 270 \sin(84°)\,\mathbf{j}$

$= 28.22\mathbf{i} + 268.52\mathbf{j}$

Let $\mathbf{b} = (320, -48°) = 320 \cos(-48°)\,\mathbf{i} + 320 \sin(-48°)\,\mathbf{j}$

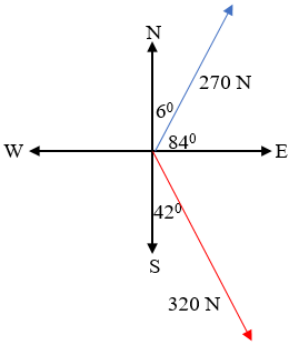
$= 214.12\mathbf{i} - 237.81\mathbf{j}$

$\mathbf{a} + \mathbf{b} = 242.34\mathbf{i} + 30.71\mathbf{j}$

angle: $\tan^{-1}\left(\frac{30.71}{242.34}\right) = 7.2°$

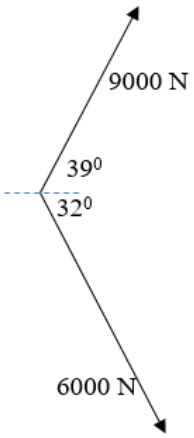
$90° - 7.2° = 82.8°$

This is N 82.8° E



Question 1

12 335 N in a direction 11.6° to the passenger's side.



Let **a** = (9000, 39°) = 9000 cos (39°) **i** + 9000 sin(39°) **j**

= 6994.31**i** + 5663.88**j**

Let **b** = (6000, –32°) = 6000 cos (–32°) **i** + 6000 sin (–32°) **j**

= 5088.29**i** – 3179.52**j**

a + **b** = 12 082.6**i** + 2484.36**j**

magnitude: $\sqrt{12\ 082.6^2 + 2484.36^2} = 12\ 335.37$

angle: $\tan^{-1}\left(\frac{2484.36}{12\ 082.6}\right) = 11.6°$

Question 2

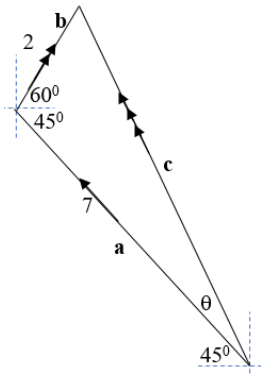
26.91 m/s 42° E

v = **b** + –**a**

$|\mathbf{v}| = |\mathbf{b} - \mathrm{a}| = \sqrt{18^2 + 20^2} = 26.91\,\mathrm{m/s}$

angle: $\tan^{-1}\left(\frac{18}{20}\right) = 42.0°$ or 042°

Question 3



magnitude 7.76 knots in direction 120.6°

Let **a** = (7, 135°) = 7 cos (135°) **i** + 7 sin (135°) **j**
= –4.95**i** + 4.95**j**

Let **b** = (2, 60°) = 2 cos (60°) **i** + 2 sin (60°) **j**
= 1**i** + 1.73**j**

Let **c** be the vector describing the speed and course.

c = **a** + **b** = –3.95**i** + 6.68**j**

magnitude of **c**: $\sqrt{(-3.95)^2 + 6.68^2} = 7.76$

$$\frac{\sin(\theta)}{2} = \frac{\sin(105°)}{7.76}$$

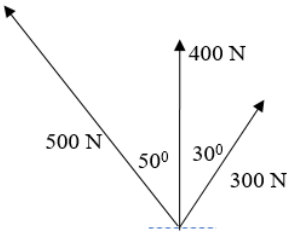
$$\sin(\theta) = 2 \times \frac{\sin(105°)}{7.76}$$

$$= 0.249$$

$$\theta = \sin^{-1}(0.249) = 14.4°$$

Required angle: 180° – (θ + 45°) = 120.6°

Question 4



Let **a** = (300, 60°) = 300 cos (60°) **i** + 300 sin (60°) **j**
= 150**i** + 259.81**j**

Let **b** = (400, 90°) = 400 cos (90°) **i** + 400 sin (90°) **j**
= 0**i** + 400**j**

Let **c** = (500, 140°) = 500 cos (140°) **i** + 500 sin (140°) **j**

= –383.02**i** + 321.39**j**

a + **b** + **c** = –233.02**i** + 981.20**j**

magnitude: $\sqrt{(-233.02)^2 + 981.20^2} = 1008.49$

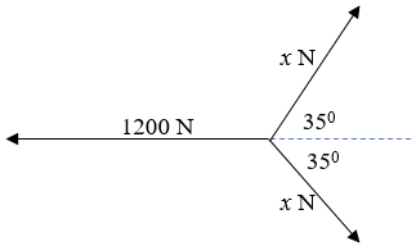
angle: $\tan^{-1}\left(\frac{981.20}{-233.02}\right) = -76.6°$

The resultant vector is in the 2nd quadrant, so the angle is 180° – 76.6° = 103.4°.

This is 103.4° – 90° = 13.4° towards the 500 N force.

Question 5

977 N



Let **a** be the first team of the winning side and let **b** be the second team of the winning side.

a = (x, 35°) = x cos (35°) **i** + x sin (35°) **j** = 0.819x**i** + 0.574x**j**

b = (x, –35°) = x cos (–35°) **i** + x sin (-35°) **j** = 0.819x**i** – 0.574x**j**

a + **b** = 1.638x**i** + 0**j**

| **a** + **b** | = |1.638x**i** + 0**j**| = 1.638x

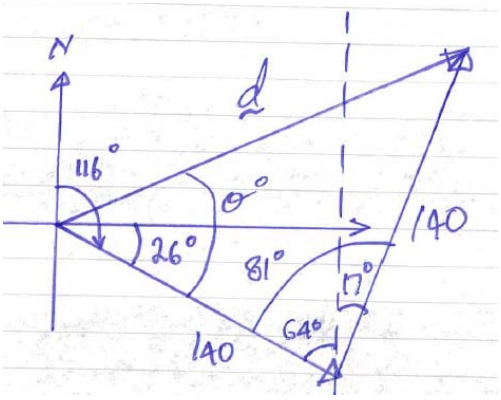
Given 1.638x = 1200 + 400

$$x = \frac{1600}{1.638} = 977$$

Question 6

181.85 knots N 66.5° E

Want $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = \mathbf{v}_2 + (-\mathbf{v}_1)$



Find the magnitude and bearing of **d**.

Apply the cosine rule

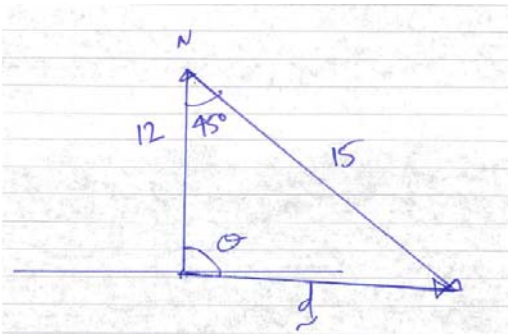
$$d^2 = 140^2 + 140^2 - 2 \times 140 \times 140 \times \cos(81°)$$
$$d = 181.85$$

Apply the sine rule

$$\frac{\sin(\theta)}{140} = \frac{\sin(81°)}{181.85}$$
$$\sin(\theta) = 140 \times \frac{\sin(81°)}{181.55}$$
$$= 0.760$$
$$\theta = \sin^{-1}(0.760) = 49.5°$$

Required angle: $116° - \theta = 116° - 49.5° = 66.5°$

Question 7



10.70 m/s N 82.5° E

Want $\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = \mathbf{v}_2 + (-\mathbf{v}_1)$

Find magnitude and bearing of **d**.

Apply the cosine rule

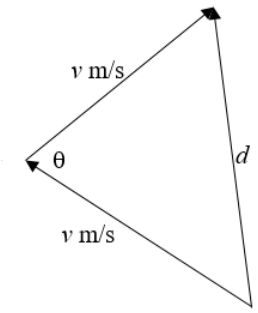
$$d^2 = 15^2 + 12^2 - 2 \times 15 \times 12 \times \cos(45°)$$
$$d = 10.70$$

Apply the sine rule

$$\frac{\sin(\theta)}{15} = \frac{\sin(45°)}{10.70}$$
$$\sin(\theta) = 15 \times \frac{\sin(45°)}{10.70}$$
$$= 0.9913$$
$$\theta = \sin^{-1}(0.9913) = 82.5°$$

Question 8

$v\sqrt{(2(1-\cos(\theta))}$ m/s

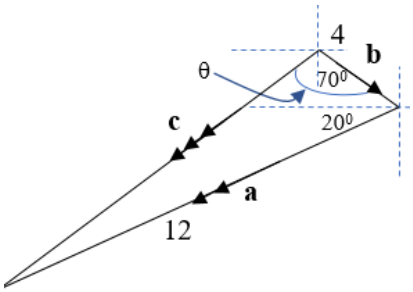


Apply the cosine rule

$$d^2 = v^2 + v^2 - 2 \times v \times v \times \cos(\theta)$$
$$= 2v^2 - 2 \times v^2 \cos(\theta)$$
$$= 2v^2\left[1 - \cos(\theta)\right]$$
$$d = \sqrt{2v^2\left[1 - \cos(\theta)\right]}$$
$$= v\sqrt{2\left[1 - \cos(\theta)\right]}$$

Question 9

magnitude 9.30 knots in direction 216°



Let **a** = (12, 200°) = 12 cos (200°) **i** + 12 sin (200°) **j**

= –11.28**i** – 4.10**j**

Let **b** = (4, –20°) = 4 cos (–20°) **i** + 4 sin (–20°) **j**

= 3.76**i** – 1.37**j**

Let **c** be the vector describing the speed and course.

c = **a** + **b** = –7.52**i** – 5.47**j**

magnitude of **c**: $\sqrt{(-7.52)^2+(-5.47)^2}=9.30$

$$\frac{\sin(\theta)}{12}=\frac{\sin(40°)}{9.30}$$

$$\sin(\theta)=12\times\frac{\sin(40°)}{9.30}$$

$$=0.829$$

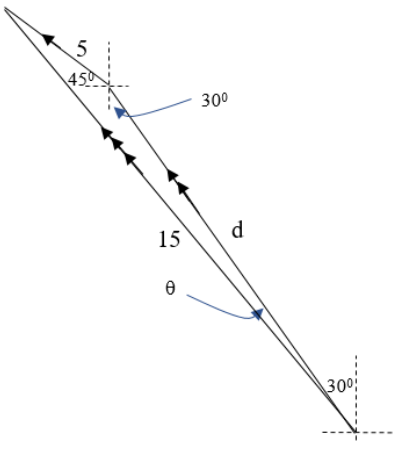
$$\theta=\sin^{-1}(0.829)=56.0°$$

Since θ is greater than 70°, take θ = 180° – 56.0° = 124.0° (the ambiguous case)

Required angle: 270° – (θ – 70°) = 216°

Question 10

magnitude 10.1 knots in direction 124.9°



$$15^2=d^2+5^2-2\times5\times d\times\cos(165°)$$

$$d^2+9.66d-200=0$$

This quadratic can be solved to obtain the positive value of d. $d=10.1$

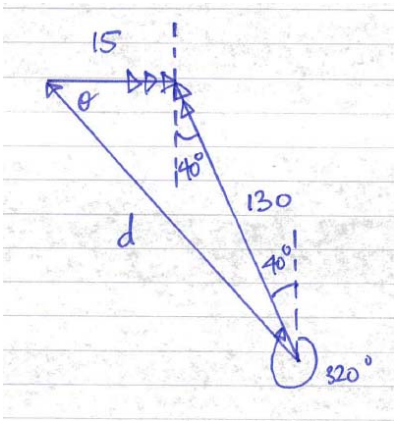
Apply the sine rule

$$\frac{\sin(\theta)}{5}=\frac{\sin(165°)}{15}$$

$$\theta=\sin^{-1}(0.772)=4.9°$$

Required angle: 90° + 30° + θ = 124.9°

Question 11



magnitude 140.11 knots in direction 315.3°

$d^2 = 15^2 + 130^2 - 2 \times 15 \times 130 \times \cos(130°)$

$d = 140.11$

Apply the sine rule

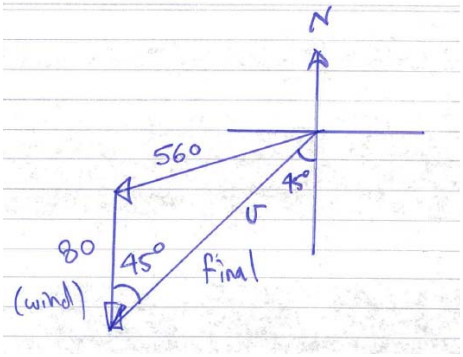
$$\frac{\sin(\theta)}{130} = \frac{\sin(130°)}{140.11}$$

$\theta = \sin^{-1}(0.711) = 45.3°$

Required angle: 360° – 40° – (180° – 130° – θ) = 315.3°

Question 12

Find the final speed the plane travels on SW bearing.



$560^2 = 80^2 + \mathbf{v}^2 - 2(80)(\mathbf{v}) \cos(45°)$

$\mathbf{v}^2 - 113.14\mathbf{v} - 307\,200 = 0$

$$v = \frac{113.14 \pm \sqrt{(113.14)^2 - 4(1)(-307\,200)}}{2(1)}$$

$$= \frac{113.14 \pm 1114.2711}{2}$$

= 613.7 km/h

(neglecting the impossible negative solution)

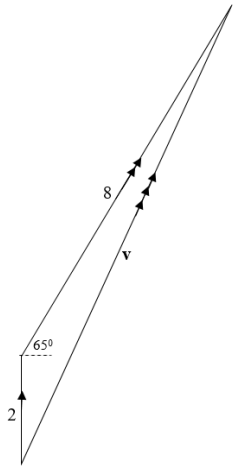
This will take $\frac{1500}{613.7}$ = 2.44 h = 2 hours 27 minutes to travel the required 1500 km.

10:05 + 2:27 = 12:32 p.m.

Question 13

9.8 knots

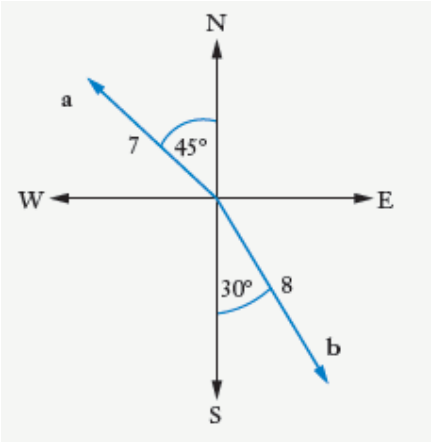
$|\mathbf{v}| = \sqrt{2^2 + 8^2 - 2 \times 2 \times 8 \times \cos(155°)} = 9.8$



Question 1

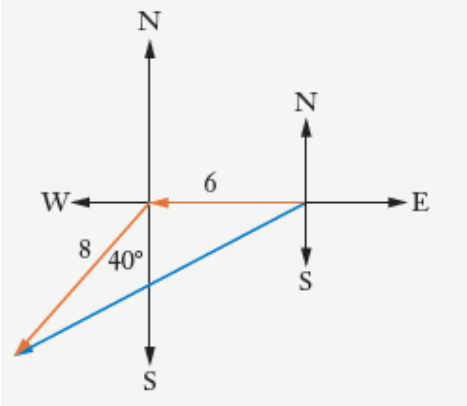
a A line representing a magnitude 7 m with direction N 45° W

b A line representing a magnitude 8 m with direction S 30° E

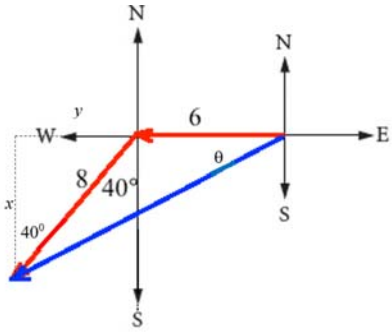


Question 2

a



b



$x = 8\cos(40°) = 6.128$

$y = 8\sin(40°) = 5.142$

$$\tan(\theta) = \frac{x}{y+6} = 0.55$$

$$\theta = \tan^{-1}(0.55) = 28.8°$$

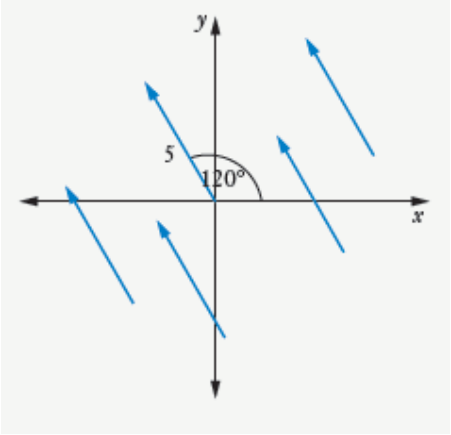
Bearing is $180° + 28.8° = 208.8°$ or $S61.2°\ W$

Question 3

a $\mathbf{p} = (5, 120°)$

polar form is (magnitude, angle made with positive x-axis)

b Any vector with the same magnitude and direction can be drawn.

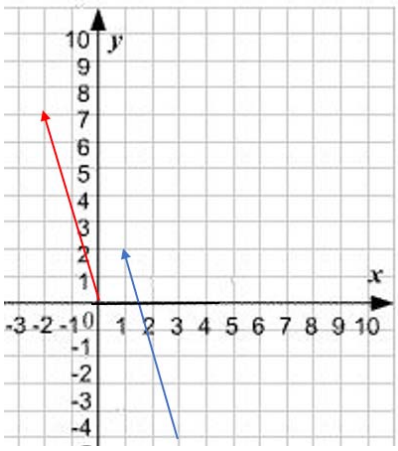


Question 4

a (–2, 6)

To translate to the origin, vector is translated 2 units left and 6 units up.

Head of vector is at $(1-3, 2-(-4)) = (-2, 6)$



b $(5-(-3), 0-(-5)) = (8, 5)$

c $(-3-1), 5-1) = (-4, 4)$

d $(0-2, 8-4) = (-2, 4)$

e $(1-(-6), -9-3) = (7, -12)$

Question 5

a $\sqrt{(24)^2+10^2} = 26$

b $\sqrt{(-3)^2+(-4)^2} = 5$

c $\sqrt{(-8)^2+15^2} = 17$

d $\sqrt{5^2+9^2} = 10.296$

e $\sqrt{10^2+12^2} = 15.621$

Question 6

a

$$\left(3\cos(30°), 3\sin(30°)\right) = \left(3 \times \frac{\sqrt{3}}{2}, 3 \times \frac{1}{2}\right)$$
$$= \left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$$

b

$$\left(8\cos(150°), 8\sin(150°)\right) = \left(-8 \times \frac{\sqrt{3}}{2}, 8 \times \frac{1}{2}\right)$$
$$= \left(-4\sqrt{3}, 4\right)$$

c

$$\left(4\cos(300°), 4\sin(300°)\right) = \left(4 \times \frac{1}{2}, -4 \times \frac{\sqrt{3}}{2}\right)$$
$$= \left(2, -2\sqrt{3}\right)$$

d

$$\left(7\cos(-45°), 7\sin(-45°)\right) = \left(7 \times \frac{1}{\sqrt{2}}, -7 \times \frac{1}{\sqrt{2}}\right)$$
$$= \left(\frac{7\sqrt{2}}{2}, -\frac{7\sqrt{2}}{2}\right)$$

e

$$\left(9\cos(230°), 9\sin(230°)\right) = \left(-5.785, -6.894\right)$$

Question 7

a (9.434, 58°)

magnitude: $\sqrt{5^2+8^2}=9.434$

angle: $\tan^{-1}\left(\frac{8}{5}\right)=58°$.

b (5, 126.9°)

magnitude: $\sqrt{(-3)^2+4^2}=5$

angle: $\tan^{-1}\left(\frac{4}{3}\right)=53.1°$

In 2nd quadrant, so required angle $180°-53.1°=126.9°$.

c (11.662, 239.0°)

magnitude: $\sqrt{(-6)^2+(-10)^2}=11.662$

angle: $\tan^{-1}\left(\frac{-10}{-6}\right)=59.0°$

In 3rd quadrant, so required angle $180°+59.0°=239.0°$.

d (9.487, 341.6°)

magnitude: $\sqrt{9^2+(-3)^2}=9.487$

angle: $\tan^{-1}\left(\frac{-3}{9}\right)=-18.43°$

In 4th quadrant, so required angle $360°-18.43°=341.6°$.

e (4, 180°)

magnitude: $\sqrt{(-4)^2+0^2}=4$

angle: $\tan^{-1}\left(\frac{0}{-4}\right)=0°$

The vector is on the negative x-axis, so the required angle is $180°$.

Question 8

a (5.831, 59.0°)

magnitude: $\sqrt{3^2+5^2}=5.831$

angle: $\tan^{-1}\left(\frac{5}{3}\right)=59.0°$.

b (6.325, 251.6°)

magnitude: $\sqrt{(-2)^2+(-6)^2}=6.325$

angle: $\tan^{-1}\left(\frac{-6}{-2}\right)=71.6°$

In 3rd quadrant, so required angle $180°+71.6°=251.6°$.

c (4.123, 284.0°)

magnitude: $\sqrt{1^2+(-4)^2}=4.123$

angle: $\tan^{-1}\left(\frac{-4}{1}\right)=-76.0°$

In 4th quadrant, so required angle $360°-76.0°=284.0°$.

d (5, 90°)

magnitude: $\sqrt{0^2+5^2}=5$

The vector lies on the positive *y*-axis, so the angle is $90°$.

e (8.246,104.0°)

magnitude: $\sqrt{(-2)^2+8^2}=8.246$

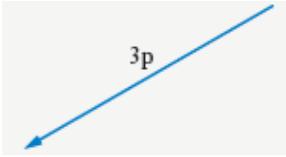
angle: $\tan^{-1}\left(\frac{8}{-2}\right)=-76.0°$

In the 2nd quadrant, so required angle $180°-76.0°=104.0°$

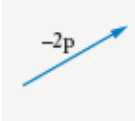
Question 9

The angle each vector makes with the horizontal should be the same as the angle **p** makes with the horizontal.

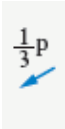
a



b



c



Question 10

a $\begin{bmatrix} 4\times 5 \\ 4\times(-4) \end{bmatrix} = \begin{bmatrix} 20 \\ -16 \end{bmatrix}$

b $\begin{bmatrix} -3\times 5 \\ -3\times(-4) \end{bmatrix} = \begin{bmatrix} -15 \\ 12 \end{bmatrix}$

c $(5\times 8, 180° + 173°) = (40, 353°)$

The negative of a vector has the same magnitude as the original but its direction is reversed (rotation of 180°).

d $(6\times 8, 173°) = (48, 173°)$

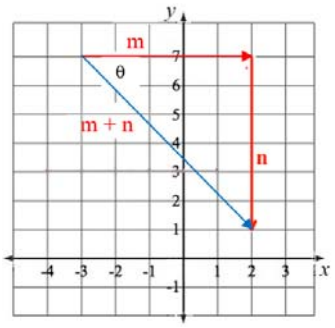
Question 11

$(7.81, 309.8°)$

magnitude: $\sqrt{(2-(-3))^2+(7-1)^2} = 7.81$

angle: $\tan^{-1}\left(\frac{6}{5}\right) = 50.2°$

Required angle $360° - 50.2° = 309.8°$



Question 12

$(18.3, 78.8°)$

Let **a** = (28, 140°) = 28 cos (140°) **i** + 28 sin (140°) **i** = –21.45**i** + 18.00**j**

Let **b** = (25, 0°) = 25 cos (0°) **i** + 25 sin (0°)**j** = 25 **i** + 0 **j**

a + **b** = 3.55**i** + 18.00**j**

magnitude: $\sqrt{3.55^2 + 18.00^2} = 18.3$

angle: $\tan^{-1}\left(\frac{18.00}{3.55}\right) = 78.8°$

So 18.3 N at 78.8° to the positive direction of the *x*-axis.

Question 13

a $(10.39, 121.1°)$

Apply the cosine rule

$d^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos(119°)$

$d = 10.39$

Apply the sine rule

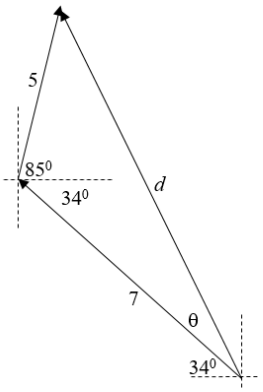
$\frac{\sin(\theta)}{5} = \frac{\sin(119°)}{d}$

$\sin(\theta) = 5 \times \frac{\sin(119°)}{10.39}$

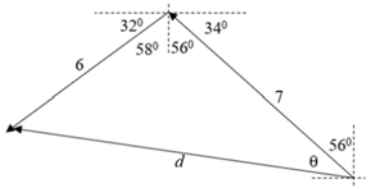
$= 0.421$

$\theta = \sin^{-1}(0.421) = 24.9°$

Required angle: $180° - (\theta + 34°) = 121.1°$



b (10.92,176.1°)



Apply the cosine rule

$d^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos(114°)$

$d = 10.92$

Apply the sine rule

$$\frac{\sin(\theta)}{6} = \frac{\sin(114°)}{d}$$

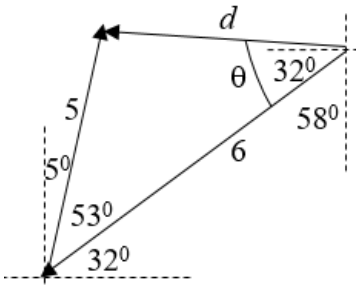
$$\sin(\theta) = 6 \times \frac{\sin(114°)}{10.92}$$

$$= 0.502$$

$\theta = \sin^{-1}(0.502) = 30.1°$

Required angle: $90° + 56° + \theta = 176.1°$

c (4.99,158.8°)



Apply the cosine rule

$d^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos(53°)$

$d = 4.99$

Apply the sine rule

$$\frac{\sin(\theta)}{5} = \frac{\sin(53°)}{d}$$

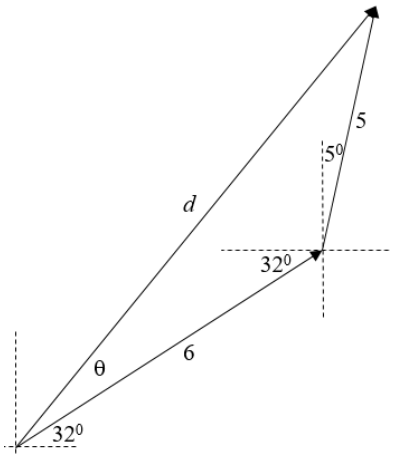
$$\sin(\theta) = 5 \times \frac{\sin(53°)}{4.99}$$

$$= 0.800$$

$\theta = \sin^{-1}(0.800) = 53.2°$

Required angle: $270° - (\theta + 58°) = 158.8°$

d $(9.85, 55.9°)$



Apply the cosine rule

$d^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos(127°)$

$d = 9.85$

Apply the sine rule

$$\frac{\sin(\theta)}{5} = \frac{\sin(127°)}{d}$$

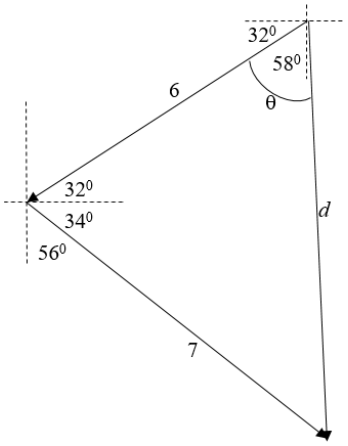
$$\sin(\theta) = 5 \times \frac{\sin(127°)}{9.85}$$

$$= 0.405$$

$\theta = \sin^{-1}(0.405) = 23.9°$

Required angle: $\theta + 32° = 55.9°$

e $(7.13, 275.8°)$



Apply the cosine rule

$d^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos(66°)$

$d = 7.13$

Apply the sine rule

$$\frac{\sin(\theta)}{7} = \frac{\sin(66°)}{d}$$

$$\sin(\theta) = 7 \times \frac{\sin(66°)}{7.13}$$

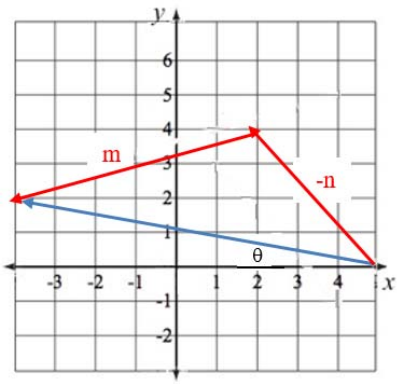
$$= 0.897$$

$\theta = \sin^{-1}(0.897) = 63.8°$

Required angle: $270° + \theta - 58° = 275.8°$

Question 14

$(9.22, 167.5°)$



Magnitude of **m** + (–**n**)

$$d^2 = \sqrt{(5-(-4))^2 + 2^2} = 9.22$$

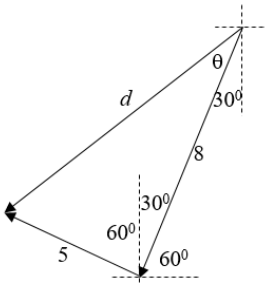
$$\tan(\theta) = \frac{2}{9}$$

$$\theta = \tan^{-1}\left(\frac{2}{9}\right) = 12.5°$$

Required angle: $180° - \theta = 167.5°$

Question 15

a $(9.43, 208°)$



Using Pythagoras' theorem

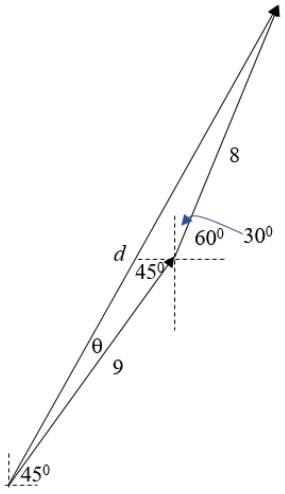
$$d^2 = 5^2 + 8^2$$
$$d = 9.433$$

$$\tan(\theta) = \frac{5}{8}$$

$$\theta = \tan^{-1}\left(\frac{5}{8}\right) = 32°$$

Required angle $180° + 60° - \theta = 208°$

b $(16.86, 52.1°)$



Apply the cosine rule

$$d^2 = 8^2 + 9^2 - 2 \times 8 \times 9 \times \cos(165°)$$
$$d = 16.86$$

Apply the sine rule

$$\frac{\sin(\theta)}{8} = \frac{\sin(165°)}{d}$$

$$\sin(\theta) = 8 \times \frac{\sin(165°)}{16.86}$$
$$= 0.123$$

$$\theta = \sin^{-1}(0.123) = 7.1°$$

Required angle $\theta + 45° = 52.1°$

Question 16

a $(0.6, 0.8)$

$\underset{\sim}{r} = 15\underset{\sim}{i} + 20\underset{\sim}{j}$

$|\underset{\sim}{r}| = \sqrt{15^2 + 20^2} = 25$

$\hat{\underset{\sim}{r}} = \dfrac{\underset{\sim}{r}}{|\underset{\sim}{r}|}$

$= \dfrac{1}{25}\left(15\underset{\sim}{i} + 20\underset{\sim}{j}\right)$

$= 0.6\underset{\sim}{i} + 0.8\underset{\sim}{j}$

b $(-0.55, 0.83)$

$\underset{\sim}{r} = -8\underset{\sim}{i} + 12\underset{\sim}{j}$

$|\underset{\sim}{r}| = \sqrt{(-8)^2 + 12^2} = 14.42$

$\hat{\underset{\sim}{r}} = \dfrac{\underset{\sim}{r}}{|\underset{\sim}{r}|}$

$= \dfrac{1}{14.42}\left(-8\underset{\sim}{i} + 12\underset{\sim}{j}\right)$

$= -0.55\underset{\sim}{i} + 0.83\underset{\sim}{j}$

Question 17

$(a, b) = a\underset{\sim}{i} + b\underset{\sim}{j}$, $\begin{bmatrix} a \\ b \end{bmatrix} = a\underset{\sim}{i} + b\underset{\sim}{j}$

a $1\underset{\sim}{i} - 3\underset{\sim}{j} = \underset{\sim}{i} - 3\underset{\sim}{j}$

b $4\underset{\sim}{i} + 5\underset{\sim}{j}$

c $-2\underset{\sim}{i} + 5\underset{\sim}{j}$

d $4\underset{\sim}{i} - 3\underset{\sim}{j}$

e $-2\underset{\sim}{i} - 5\underset{\sim}{j}$

f $6\cos(56°)\underset{\sim}{i} + 6\sin(56°)\underset{\sim}{j} = 3.36\underset{\sim}{i} + 4.97\underset{\sim}{j}$

g $8\cos(200°)\underset{\sim}{i} + 8\sin(200°)\underset{\sim}{j} = -7.52\underset{\sim}{i} - 2.74\underset{\sim}{j}$

h $3\cos(30°)\underset{\sim}{i} + 3\sin(30°)\underset{\sim}{j} = 3 \times \dfrac{\sqrt{3}}{2}\underset{\sim}{i} + 3 \times \dfrac{1}{2}\underset{\sim}{j} = \dfrac{3\sqrt{3}}{2}\underset{\sim}{i} + \dfrac{3}{2}\underset{\sim}{j}$

i $10\cos(142°)\underset{\sim}{i} + 10\sin(142°)\underset{\sim}{j} = -7.88\underset{\sim}{i} + 6.16j$

j $5\cos(120°)\underset{\sim}{i} + 5\sin(120°)\underset{\sim}{j} = 5 \times \left(-\dfrac{1}{2}\right)\underset{\sim}{i} + 5 \times \dfrac{\sqrt{3}}{2}\underset{\sim}{j} = -\dfrac{5}{2}\underset{\sim}{i} + \dfrac{5\sqrt{3}}{2}\underset{\sim}{j}$

Question 18

a $(-6 + 9, 11 + -8) = (3, 3)$

b $\begin{bmatrix} 6 + 11 \\ -7 + -3 \end{bmatrix} = \begin{bmatrix} 17 \\ -10 \end{bmatrix}$

c $(10 + -4)\underset{\sim}{i} + (-5 + 11)\underset{\sim}{j} = 6\underset{\sim}{i} + 6\underset{\sim}{j}$

Question 19

a $(3+-1, 5+4) = (2, 9)$

b $(-2-3, -3-5) = (-5, -8)$

c $(3\times(-2), 3\times(-3)) + (2\times(-1), 2\times 4) = (-6, -9) + (-2, 8)$
$= (-8, -1)$

d $(5\times 3, 5\times 5) + (-3\times(-2), -3\times(-3)) = (15, 25) + (6, 9)$
$= (21, 34)$

e $(3+(-2)+(-1), 5+(-3)+4) = (0, 6)$

Question 20

a $\begin{bmatrix} -2-4 \\ 3-(-1) \end{bmatrix} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$

b $\begin{bmatrix} 2\times(-2) \\ 2\times 3 \end{bmatrix} + \begin{bmatrix} 3\times 4 \\ 3\times(-1) \end{bmatrix} = \begin{bmatrix} -4+12 \\ 6-3 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$

c $\begin{bmatrix} 4 \\ -1 \end{bmatrix} + \begin{bmatrix} 3\times 1 \\ 3\times 3 \end{bmatrix} = \begin{bmatrix} 4+3 \\ -1+9 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$

d $\begin{bmatrix} 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 2\times(-2) \\ 2\times 3 \end{bmatrix} = \begin{bmatrix} 1+4 \\ 3-6 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

e $\begin{bmatrix} -2+1 \\ 3+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

Question 21

$(a,b) = a\underset{\sim}{i} + b\underset{\sim}{j}$, $\begin{pmatrix} a \\ b \end{pmatrix} = a\underset{\sim}{i} + b\underset{\sim}{j}$

a $-4\underset{\sim}{i} + 3\underset{\sim}{j}$

b $\frac{1}{8}\underset{\sim}{i} + \frac{-5}{7}\underset{\sim}{j} = \frac{1}{8}\underset{\sim}{i} - \frac{5}{7}\underset{\sim}{j}$

c $3\sqrt{5}\underset{\sim}{i} + \sqrt{6}\underset{\sim}{j}$

d $17\cos(38°)\underset{\sim}{i} + 17\sin(38°)\underset{\sim}{j} = 13.40\underset{\sim}{i} + 10.47j$

e $11\cos(251°)\underset{\sim}{i} + 11\sin(251°)\underset{\sim}{j} = -3.58\underset{\sim}{i} - 10.40j$

Question 22

(3.65,36.88), (37.06, 84.35°)

n = $22\cos(180° - 39°)\underset{\sim}{i} + 22\cos(180° - 39°)\underset{\sim}{j} = -17.10\underset{\sim}{i} + 13.85j$

m = $31\cos(48°)\underset{\sim}{i} + 31\sin(48°)\underset{\sim}{j} = 20.74\underset{\sim}{i} + 23.04j$

n + **m** = $3.65\underset{\sim}{i} + 36.88j$

magnitude: $\sqrt{3.65^2 + 36.88^2} = 37.06$

$\theta = \tan^{-1}\left(\frac{36.88}{3.65}\right) = \tan^{-1}(10.104) = 84.35°$

Question 23

(67.19, 311.5°)

a = $31\cos(180° - 38°)\underset{\sim}{i} + 31\sin(180° - 38°)\underset{\sim}{j} = 31\cos(142°)\underset{\sim}{i} + 31\sin(142°)\underset{\sim}{j}$

b = $58\cos(180° + 76°)\underset{\sim}{i} + 58\sin(180° + 76°)\underset{\sim}{j} = 58\cos(256°)\underset{\sim}{i} + 58\sin(256°)\underset{\sim}{j}$

c = $84\cos(270° + 81°)\underset{\sim}{i} + 84\sin(270° + 81°)\underset{\sim}{j} = 84\cos(351°)\underset{\sim}{i} + 84\sin(351°)\underset{\sim}{j}$

$\mathbf{a} + \mathbf{b} + \mathbf{c} = \left(31\cos(142°) + 58\cos(256°) + 84\cos(351°)\right)\underset{\sim}{i}$

$+ \left(31\sin(142°) + 58\sin(256°) + 84\sin(351°)\right)\underset{\sim}{j}$

$= 44.51\underset{\sim}{i} - 50.33\underset{\sim}{j}$

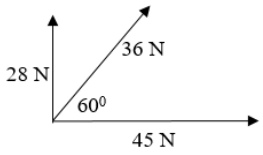
$|\mathbf{a} + \mathbf{b} + \mathbf{c}| = \sqrt{44.51^2 + (-50.33)^2} = 67.19$

angle: $\tan^{-1}\left(\frac{-50.33}{44.51}\right) = -48.5°$

In the 4th quadrant, the required angle is $360° - 48.5° = 311.5°$

Question 24

(86.44, 43.2°)



$\mathbf{a} = 28\cos(90°)\underset{\sim}{i} + 28\sin(90°)\underset{\sim}{j} = 28\underset{\sim}{j}$

$\mathbf{b} = 36\cos(60°)\underset{\sim}{i} + 36\sin(60°)\underset{\sim}{j} = 18\underset{\sim}{i} + 31.18\underset{\sim}{j}$

$\mathbf{c} = 45\cos(0°)\underset{\sim}{i} + 45\sin(0°)\underset{\sim}{j} = 45\underset{\sim}{i}$

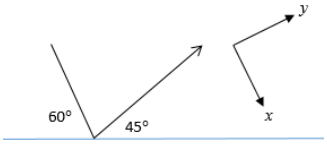
$\mathbf{a} + \mathbf{b} + \mathbf{c} = 63\underset{\sim}{i} + 59.18\underset{\sim}{j}$

$|\mathbf{a} + \mathbf{b} + \mathbf{c}| = \sqrt{63^2 + 59.18^2} \approx 86.44$

angle: $\tan^{-1}\left(\frac{59.18}{63}\right) = 43.2°$

86.44 N at 43.2° to the horizontal.

Question 25



Original velocity = 0.3 **i**
Final velocity = –0.2 cos (75°) **i** + 0.2 sin (75°) **j**
Change in velocity = –(0.2 cos (75°) + 0.3) **i** + 0.2 sin (75°) **j**
= –0.3517…**i** + 0.1931…**j**

Magnitude = $\sqrt{(-0.3517...)^2 + (0.1931...)^2}$ = 0.4013… m/s

Direction: $\tan(\theta) = \frac{0.1931...}{-0.3517...}$ (2nd quadrant)

$\theta = 180° - 28.7751…° = 151.2248…°$

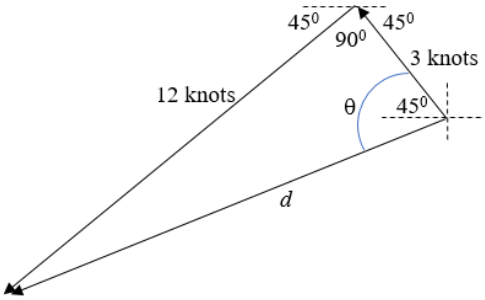
Question 26

12.37 knots at 239°
Magnitude

$d = \sqrt{3^2 + 12^2} = 12.37$

Angle: $\tan(\theta) = \frac{12}{3}$, $\theta = 76°$

required angle: $270° - (\theta - 45°) = 239°$

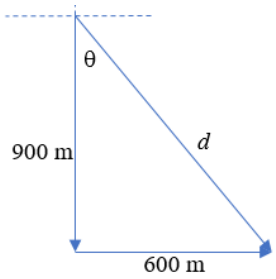


Question 27

$(1082, 303.7°)$

$d = \sqrt{600^2 + 900^2} = 1081.67 \approx 1082 \text{ m}$

$\tan^{-1}\left(\frac{600}{900}\right) = 33.7°$



Displacement = 900 m vector + 600 m vector, puts angle in 4th quadrant.

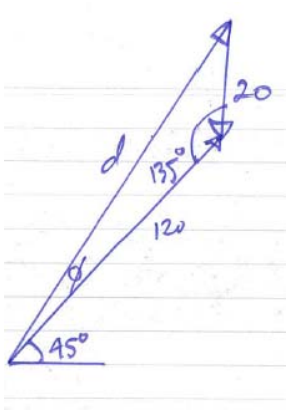
required angle = 270° + 33.7° = 303.7°.

Question 28

134.9 knots at $39.0°$

Magnitude of speed required

$d = \sqrt{20^2 + 120^2 - 2 \times 20 \times 120 \times \cos(135°)} \approx 134.9$



angle

$$\frac{\sin(\theta)}{20} = \frac{\sin(135°)}{134.89}$$

$$\theta = \sin^{-1}\left[20 \times \frac{\sin(135°)}{1034.89}\right]$$

$$= 6.01°$$

required angle: $90° - 45° - \theta = 39.0°$